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Top income share and economic growth: Linear and non-linear effects

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Abstract

This paper estimates the impact of inequality on growth, focusing on non-linearity and using top income share data that provides us with yearly observations over a long time period. A first result is that the relationship between inequality and growth is non-linear in contrast with most existing studies which focus on a linear relationship. Using panel threshold regression and panel smooth threshold regression methods, we show that the impact of inequality on growth is negative but the effect is larger when inequality is low. This result differs from existing work that stresses either a positive effect using linear estimation or a concave relationship using simple non-linearity. Lastly, heterogeneity across countries is related to the magnitude and the timing of the change in the elasticity.

Keywords: Inequality, growth, top income share, panel threshold regression

JEL classification: E24; N1

1 Introduction

This paper revisits the empirical literature assessing the impact of inequality on growth testing for non-linearities. This is in contrast with the existing literature, which has almost exclusively focused on linear estimation (Banerjee and Duflo 2003 is a noticeable exception). The main reason is that Gini indexes, the most popular inequality proxy, are only available on an annual basis since the early 1980s for high income countries.[1] As a result, in an attempt to cover the period anterior to the 1980s, the majority of existing papers have built panel database by taking five years averages. The low frequency of these data sets has had implications for the type of estimation used.

Contrastingly, in this paper we are using two panels with a long time dimension, which is made possible by using top income share as a proxy for inequality. A first panel covers the period 1950-2010 for 11 high income countries. A second panel covers the period 1920-2010 for five high income countries.[2] As a consequence, we are able to use more advanced panel estimation techniques such as panel threshold regression and panel smooth transition regression to test for the importance of non-linearity.

A first result is that the relationship between inequality and growth is best described by non-linearities. Further, panel threshold estimation concludes that the relationship between top income share and growth is negative where traditional (linear) fixed effects panel regression with similar country and time coverage finds a small positive relationship as in Andrews et al. (2011). We identify high- and low- inequality regimes. The relationship between inequality and growth is negative in both regimes but the size of the effect is larger when inequality is relatively small.

This result stands in contrast with existing work such as that of Banerjee and Duflo (2003), who establish a concave relationship by either introducing a square term in the regression or by using piecewise regression where the choice of the cut-off point is exogenous. The advantage of panel threshold regression over these methods lies in the endogenous choice of the threshold. Lastly, panel smooth transition regression confirms that the relationship is negative and convex but also points to large heterogeneity across countries with respect to the timing of changes in the elasticities and the magnitude of the change.

The empirical literature assessing the importance of inequality on growth is closely tied to the improvement in the data. Research based on cross sectional databases often found a negative impact of inequality such as Alesina and Rodrik (1994). The development of longitudinal data with a small time dimension brought on the opposite conclusion, i.e. that inequality enhances growth (Forbes, 2000). This led economists to argue that the sign of the effect depends on the time dimension adopted, positive in the short-term and negative in the long-term (Halter et al., 2014).[3]

Closer to our concern, Banerjee and Duflo (2003) have argued that linear panel estimation are misleading given that the relationship between inequality and growth is strongly non-linear. They stress the importance of changes in inequality that reduce growth irrespective of the direction of the change as well as the concave relationship between inequality level and growth. Their main limitation is the small time dimension of the database that limits the statistical power of the tests.

The availability of data on top income shares has opened up new avenues to revisit the inequality and growth debate by providing economists with annual observations over long period of time. The degree

[1] See for instance the World Wealth and Income Database 3.3.

[2] The first panel contains 11 countries: Australia, Canada, Denmark, Finland, France, Japan, New Zealand, Norway, Sweden, the United Kingdom, and the United States. The five countries of the second panel are: Australia, Canada, France, Japan and the United States

[3] Berg et al. 2012 show that inequality reduces the duration of growth spells.

of complementarity and/or substitution between top income share and the Gini coefficient is open to debate. Voitchovsky (2005) and Cingano (2014) argue that changes at the top of the distribution and at the bottom of the distribution have opposite effects on economic growth and that one indicator is not a substitute for the other. In contrast, Leigh (2007) points out that top income shares closely track Gini indexes. Two main studies have relied on top income share to assess the impact of inequality on growth. Andrews et al. (2011) conclude towards a small and positive impact of inequality on growth using fixed effects panel regressions for 12 countries over the period 1960-2000. In contrast, Herzer and Vollmer (2013) find a negative relationship using cointegration for a panel of 9 countries over the period 1960-1996.

The rest of the paper is organized as follows. Section 2 describes the trends in top income share and growth. Section 3 presents the linear fixed effects panel regression. This section in particular reproduces the results of Andrews et al. (2011) as well as the simple nonlinear experiment of Banerjee and Duflo (2003). Section 4 presents our evidence on a negative and convex relationship using panel threshold regression. Section 5 brings further evidence about the negative and convex relationship pointing as well to the heterogeneity across countries using panel smooth transition regression. A final section concludes.

2 Trends in top income share and growth

Figure 1(a) displays the average evolution of the top 1 percent income share and real per capita GDP growth over the period 1950-2010 for a panel of 11 countries. Data for real GDP per capita comes from the Maddison Project[4] and the top 1 percent income share is taken from the World Wealth and Income Database. The average evolution is obtained by fitting a fixed effect regression of the type $y_{i,t} = c_i + d_t + \epsilon_{i,t}$ with $y_{i,t}$ either the difference of the log real GDP per capita or the top 1 percent income share, c_i country fixed effects and d_t year fixed effects. The smoothed real per capita GDP growth is obtained in a similar way after filtering historical real per capita GDP growth using frequency domain filters keeping the low frequency.[5]

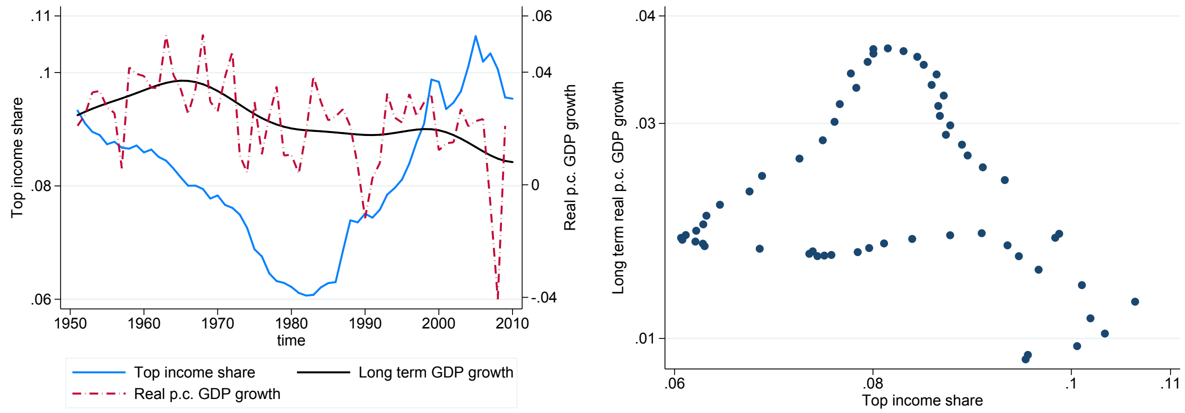
The average evolution of the top income share shows two periods. From 1950 to 1981, the top income share declines at an average of -0.31 percentage points per year. After 1981, the top 1 percent income share increases at an average of 0.11 percentage points per year. The increase in the top income share is associated with a slowdown of economic growth. Growth averages 2.7 percent a year over the period 1950-1981. Since 1981, growth falls to 1.7 percent annually. The slowdown of economic growth appears more clearly when looking at the smoothed series. The local maximum of long term growth after 1981 corresponds to the local minimum of long term growth before 1981.

The scatter plot between top income share (on x-axis) and the smoothed real GDP per capita growth (on the y-axis) in Figure 1(b) shows a rather non-linear relationship. The relationship is either negative or flat except for the period 1966 early 1980's where the relationship is positive. In fact, the correlation between the top income share and long-term growth changes sign from positive (0.53) before 1981 to negative (-0.56) after that year. Similarly, simple regressions of the type $y_t = c + x_t + \epsilon_t$ with y_t the points corresponding to the year fixed effects of the regression for long term growth and x_t the points corresponding to the year fixed effects for the regression for top income share show a positive (and

[4] http://www.ggdc.net/maddison/maddison-project/home.htm

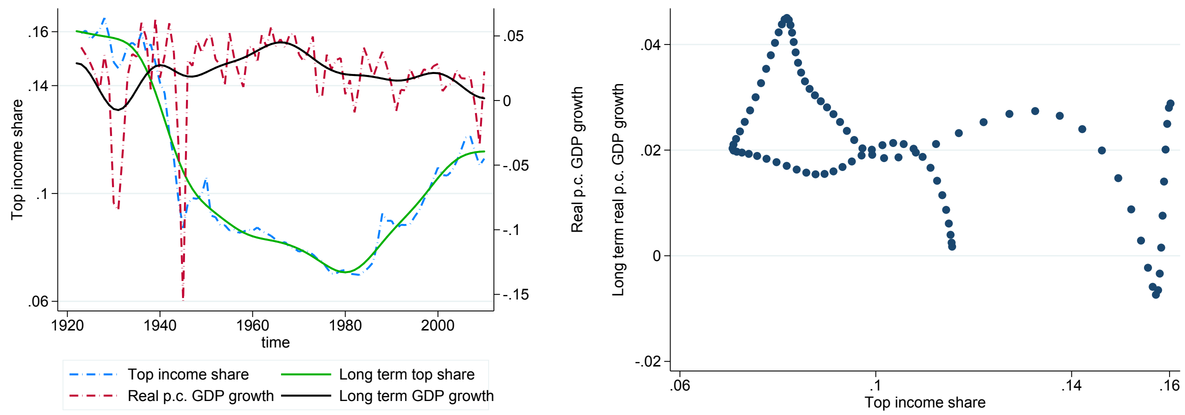
[5] The smoothed series are obtained using wavelet filters based on historical time series from 1871 to 2010 and keeping the low frequency series corresponding to the 32 years and beyond frequency. The use of wavelet is motivated by the shortcoming of using HP filters on annual data.

Figure 1: Top income share and growth - 1950-2010 - 11 countries



These figures report the average top income share and the average difference of the log of GDP per capita for a panel of 11 countries over 1950-2010. The sample of countries is: Australia, Canada, Denmark, Finland, France, Japan, New-Zealand, Norway, Sweden, United-Kingdom, and United-States. The figure is made of the year fixed effect from the regressions: $y_{i,t} = c_i + d_t + \epsilon_{i,t}$ with $y_{i,t}$ either the difference of the log real GDP per capita or the top 1 percent income share, c_i country fixed effects and d_t year fixed effects.

Figure 2: Top income share and growth - 1922-2010 - 5 countries



These figures report the average top income share and the average difference of the log of GDP per capita for a panel of five countries over 1922-2010. The sample of countries is: Australia, Canada, France, Japan and the United-States. The figure is made of the year fixed effect from the regressions: $y_{i,t} = c_i + d_t + \epsilon_{i,t}$ with $y_{i,t}$ either the difference of the log real GDP per capita or the top 1 percent income share, c_i country fixed effects and d_t year fixed effects.

significant) coefficient (0.31) in the first part of the sample that turns negative (-0.13 and significant) in the second part of the sample.

Figures 2(a) and 2(b) display top income share and growth over a longer time frame 1921-2010 and for a smaller subset of countries (Australia, Canada, France, Japan and the United States). The post World War II period is similar to the 11 countries panel with i) growth falling after 1981 when top income share increases ii) a negative relationship between growth and top income share except for the period going from the mid 1960's to the early 1980's. Looking at the beginning of the period, the 1920's and 1930's are characterised by a positive relationship between top income share and growth. In contrast, declining inequality is associated with growth trending up over the period 1940-1965. The visual impression is that the non-linearity seems more important in Figure 1 (b) than in Figure 2 (b).

3 Linear regression and simple non-linearity

In section 3.1 we discuss the main issues related to the estimation of a dynamic growth model. In section 3.2 we present the results of our linear estimation and in section 3.3 we discuss the result of simple nonlinearity as proposed by Banerjee and Duflo (2003).

3.1 Estimation strategy

Assessing the impact of the top income share on growth is done by estimating an annual panel data growth model with a lagged income variable to control for convergence.

$$Y_{i,t} - Y_{i,t-1} = (\alpha_i - 1)\, Y_{i,t-1} + \beta_x x_{i,t-1} + \beta_z Z_{i,t-1} + \epsilon_{i,t} \tag{1}$$

with $t-1$ the observation from the preceding year and i represents a particular country. $Y_{i,t}$ is the log of real GDP per capita. $\epsilon_{i,t}$ includes a country specific effect n_i, a time-specific effect h_t and an error term v_{it}. The explanatory variable is $x_{i,t-1}$ the lagged top 1 percent income share. $Z_{i,t-1}$ is a vector of lagged control variables. The model considered is dynamic and can be rewritten as:

$$Y_{i,t} = \alpha Y_{i,t-1} + \beta_x x_{i,t-1} + \beta_z Z_{i,t-1} + \epsilon_{i,t} \tag{2}$$

The dynamic nature of the model can potentially generate a biased estimate of the coefficients α. The bias arises as a result of the correlation between the lagged dependent variable and the country fixed effects. Another bias can be related to the endogeneity of the explanatory variables. A last issue is related to omitted variables. Their implications for the quality of the estimation has produced a large literature in the context of growth regression (see for instance Sala-I-Martin 1997).

In the literature, the endogeneity issue is addressed by using the first-difference GMM technique, in particular the estimator developed by Arellano and Bond (1991). Taking the first difference of the equation removes the time invariant effect n_i. In addition, instruments are constructed by taking lagged values of the explanatory variable in level to proxy for the first difference of the explanatory variables. A variant of the first-difference GMM estimator is the system GMM estimator developed by Arellano and Bover (1995) and Blundell and Bond (1998). This variant has the characteristic of retaining the cross-country information that the first difference eliminates. The system GMM estimator has a more complex set of instruments, which is made of both the variables in first difference and the variables in level.

Note that the system GMM estimator is a natural solution for panels with a small time dimension. With respect to panel with a long time dimension, the number of instruments in the GMM estimator increases with the time dimension of the data. Interestingly, an advantage of panel with a long time dimension is that biases associated with the lagged dependent variable tend to disappear as put forward by Nickell (1981). In fact, it is shown that the bias in a dynamic panel is a decreasing function of the number of time period.

3.2 Linear estimation

In this section that focuses on linear effects, we estimate equation 3. The dependent variable is the yearly difference of the log of GDP per capita $y_{i,t}$ of country i at time t. The explanatory variable is $x_{i,t-1}$ the lagged top 1 percent income share. $z_{i,t-1}$ is a vector of additional control variables that includes the log of GDP per capita lagged one period. $\epsilon_{i,t}$ includes country and year fixed effects as well as an error term.

$$y_{i,t} = c + \beta_x x_{i,t-1} + \beta_z z_{i,t-1} + \epsilon_{i,t} \tag{3}$$

We deal with the omitted variable issue in two ways. First we take as the dependent variable the difference of the log of GDP per capita. It follows that the estimates are no longer biased by any omitted variables that are constant over time. Secondly, the choice of control variables $z_{i,t-1}$ are based on those variables that pass the *extreme-bonds test* as discussed in Sala-I-Martin (1997). These control variables include the log of GDP per capita in order to control for the fact that economic growth varies inversely with the level of economic development, a measure of human capital as the improvement of skills is one of the main source of economic growth and the ratio of investment to GDP. We also add the rate of inflation to control for macroeconomic stability (see Barro 2000 for a detailed discussion) in order for our control variables to be identical those used in recent papers such as Halter et al. (2014) and Andrews et al. (2011).[6]

All the control variables enter the regression with a lag. Human capital and investment-to-GDP ratios are taken from the Penn World Tables. Human capital is measured by the average years of schooling, taken from Barro and Lee (2010). Regarding the long panel 1920-2010, investment to GDP and inflation are taken primarily from Schularick and Taylor (2012). Missing points corresponding to World War II have been interpolated linearly using Piketty and Zucman (2014).

Equation 3 is estimated using OLS with country and time fixed effects. There are two motivations for this. The first is that we want to have a point of comparison with Andrews et al. (2011) that use a similar estimation procedure arguing that “any lagged dependent variable bias arising from the inclusion of lagged GDP per capita approaches zero as the number of time periods approaches infinity”. This point has been shown by Nickell (1981). The second is that the linear fixed effect model serves as a basis of comparison with the non-linear estimation. Panel threshold regression and panel smooth transition regression cannot be combined with GMM estimator in the current state of the literature.

[6] One way around the problem of omitted variables bias is the general to specific procedure put forward by Hendry and Krolzig (2004).

Lastly, equation 3 is estimated for two panels. The first panel covers the period 1950-2010 and includes 11 high income countries: Australia, Canada, Denmark, Finland, France, Japan, New Zealand, Norway, Sweden, the United Kingdom, and the United States in line with Andrews et al. (2011).[7] The second panel covers the period 1920-2010 and includes five countries: Australia, Canada, France, Japan and the United States.

Table 1 column 1 replicates the results of Andrews et al. (2011), which constitute the main attempt to test the impact of the top income share on growth using panel regressions over the period 1960-2000. The estimation includes country and time fixed effects and the standard errors allow for intra group correlation. The sign associated with the top 1 percent income share for a panel of 11 countries over the period 1960-2000 is positive and small (0.177) in line with the results reported in Andrews et al. (2011).[8] The coefficient is robust to changes in the period considered. The coefficient is 0.171 over the period 1950-2000, 0.11 over the period 1960-2000 and 0.1 over the period 1950-2010.[9] As argued by Andrews et al. (2011): “There appears to be some trickledown effect in the long run, but because the impact of a change in inequality on economic growth is quite small, it is difficult to be sure from our estimates whether the bottom 90 per cent will really be better off or not”. However, excluding countries such as France, Japan or excluding extractive countries such as Australia, Canada and Norway produces a coefficient not statistically different from zero (see Table A2 in the appendix).

Lastly, the control variables have the expected signs regardless of the inclusion of time fixed effects. The log of GDP per capita has a negative sign in line with the fact that countries with the lowest GDP per capita have the highest growth potential. Human capital is significant and negative. The sign may depend on the measurement of human capital: average years of schooling, primary attaintment or secondary attaintment (see Barro 2000). In addition, human capital is an annual interpolation of data available on a five years average contrastingly to the other control variables, which are available on an annual basis. The investment-to-GDP ratio has a negative coefficient, however the sign of the effect is sensitive to the lag structure. The coefficient turns positive if the investment to GDP ratio enters the equation contemporaneously. Lastly, the rate of inflation is associated with a negative sign, which indicates that price instability is detrimental to growth.

3.3 Simple non-linear estimation

Banerjee and Duflo (2003) explain the heterogeneity of results of existing studies[10] by the fact that they try to capture a complex and non-linear relationship with a linear estimation. Banerjee and Duflo (2003) test non-linearity by estimating two equations. Equation 4 introduces the change in inequality as well as its square term along the level of inequality. Equation 5 introduces the squared level of inequality.

$$y_{i,t} = \alpha_i + \beta_x x_{i,t-1} + \beta_{dx}\left(x_{i,t-1} - x_{i,t-2}\right) + \beta_{dx2}\left(x_{i,t-1} - x_{i,t-2}\right)^2 + \beta_z z_{i,t-1} + \epsilon_{i,t} \quad (4)$$

$$y_{i,t} = \alpha_i + \beta_x x_{i,t-1} + \beta_{x2} x_{i,t-1}^2 + \beta_z z_{i,t-1} + \epsilon_{i,t} \quad (5)$$

Using a 5-year average panel of the Gini index for high- and middle- income countries, Banerjee and Duflo (2003) find that the linear term is insignificant while the quadratic term is negative and significant in equation 4. The non-linearity is further tested by using a Kernel estimation. The Kernel indicates that

7 We do not include Switzerland as top income shares are only available every two years.
8 We excluded Switzerland from our panel as data are available only every two years.
9 See Table A2 in appendix
10 Using Gini indexes as a measure of inequality

Table 1: Linear regression and simple non-linearity

ID	11	11	11	11
	1960-2000	1950-2010	1950-2010	1950-2010
	linear	linear	square	piecewise
$Top1_{t-1}$	0.177**	0.100*	0.217***	
	(0.071)	(0.052)	(0.063)	
$Top1^2_{t-1}$			-4.810***	
			(1.464)	
$Top1_{t-1} < \gamma$				0.272***
				(0.073)
$Top1_{t-1} > \gamma$				-0.314**
				(0.114)
$lgdppk_{t-1}$	-0.064***	-0.042***	-0.043***	-0.043***
	(0.008)	(0.004)	(0.004)	(0.004)
hc_{t-1}	-0.020**	-0.020***	-0.020***	-0.020***
	(0.009)	(0.005)	(0.005)	(0.005)
$\left(\frac{i}{y}\right)_{t-1}$	-0.135***	-0.091***	-0.099***	-0.097***
	(0.038)	(0.027)	(0.027)	(0.028)
π_{t-1}	-0.015	-0.012	-0.000	-0.000
	(0.009)	(0.007)	(0.000)	(0.000)
_cons	0.692***	0.462***	0.476***	0.479***
	(0.084)	(0.039)	(0.043)	(0.043)
N	451	660	660	660
R^2	0.496	0.493	0.501	0.503
$CountryFE$	Yes	Yes	Yes	Yes
$YearFE$	Yes	Yes	Yes	Yes

Standard errors in parentheses * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. This table summarizes the results of the country and time fixed effects regressions for a panel of 11 countries over the period 1950-2010: Australia, Canada, Denmark, Finland, France, Japan, New Zealand, Norway, Sweden, the United Kingdom, and the United States. The dependent variable is the difference of the log of GDP per capita in 1990 international GK $. The exogenous variable is the lagged top 1 percent income share. The control variables are the lagged GDP per capita in log, lagged human capital, lagged investment to GDP ratio, and lagged inflation rate. The standard errors allow for cross-section correlation.

Table 2: Linear regression and simple non-linearity

	(1)	(2)	(3)	(4)	(5)
ID	5	5	5	5	5
	1950-2010	1950-2010	1950-2010	1921-2010	1921-2010
	linear	square	piecewise	linear	square
$Top1_{t-1}$	0.210*	0.213**		-0.304**	-0.370
	(0.086)	(0.076)		(0.110)	(0.295)
$Top1^2_{t-1}$		-5.595**			1.610
		(1.513)			(4.927)
$Top1_{t-1} < \gamma$			0.318**		
			(0.085)		
$Top1_{t-1} > \gamma$			-0.839**		
			(0.291)		
$lgdppk_{t-1}$	-0.067***	-0.066***	-0.065***	-0.038	-0.037
	(0.002)	(0.002)	(0.002)	(0.030)	(0.027)
$\left(\frac{i}{y}\right)_{t-1}$	0.124	0.095	0.100	0.070	0.074
	(0.068)	(0.090)	(0.080)	(0.461)	(0.468)
π_{t-1}	-0.190**	-0.177**	-0.183**	-0.014	-0.014
	(0.043)	(0.040)	(0.047)	(0.073)	(0.075)
_cons	0.598***	0.616***	0.609***	0.415	0.367
	(0.033)	(0.013)	(0.019)	(0.206)	(0.172)
N	305	305	305	445	445
R^2	0.676	0.685	0.683	0.302	0.303

Standard errors in parentheses * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. This table summarizes the results of the country and time fixed effects regressions for a panel of five countries over the period 1921-2010: Australia, Canada, France, Japan and the United States. The dependent variable is the difference of the log of GDP per capita in 1990 international GK \$. The exogenous variable is the lagged top 1 percent income share. The control variables are the lagged GDP per capita in log, lagged investment to GDP ratio, and lagged inflation rate. The standard errors allow for cross-section correlation.

the relationship has the shape of an inverted U when considering change in inequality. This points to the negative impact of changes in inequality in either direction. Further, relying on a piecewise regression, an increase in inequality is associated with a negative coefficient, while a decrease in inequality is associated with a positive coefficient. Turning to equation 5, the squared level of inequality appears negative and significant. The Kernel regression shows a U-shaped relationship, the correlation with growth being negative(positive) above(below) a certain value of the Gini coefficient.

The small time series dimension inherent to Gini databases limits the power of the tests and the power of the estimation. Taking advantage of the long time dimension of our panel, we implement an experiment similar to Banerjee and Duflo (2003) introducing square terms in the regression according to equation 4 and equation 5. Table 1 reproduces the regression with squared inequality level (column 4) and the piecewise regressions (column 5) corresponding to equation 5. The estimation of equation 4 is not reproduced as the coefficients associated with the change in inequality are insignificant. In column 4, the sign of the inequality level is positive (0.217) and significant, while the sign of the inequality squared is negative (-4.8) and significant. In this regression, the top income share is standardized with its average. It follows that the overall impact of inequality on growth is given by $\frac{\partial y_{i,t}}{\partial x_{i,t-1}} = 0.217 - 2 * 4.810 x_{i,t-1}$. This relationship describes an inverted U shape with a maximum corresponding to a standardized top income share equal to 0.022 $\left(\frac{\partial y_{i,t}}{\partial x_{i,t-1}} = 0\right)$. The relationship between inequality and growth is concave. Inequality has a positive impact on growth when standardized top income share is below 0.022 and a negative impact beyond that point.

The piecewise regression in column 5 confirms this result. For the purpose of the piecewise regression, the standardized top income share is split into two variables according to the cut-off point 0.022. The sign associated with inequality is positive (0.272) under the cut-off point and the sign turns negative (-0.314) above the cut-off point. The number of points above the cut-off point is rather small with 79 relative to the size of the sample (660 observations). In those countries where the top income share follows a U-shaped pattern – as for example in Canada, the United Kingdom or the United States - the data above the cut-off points include in particular the past two decades, the years 1990s and 2000s.

Historical times series for the top income share enable us to look at the link between inequality and growth over a long time period from 1921 to 2010 for a smaller subset of countries: Australia, Canada, France, Japan and the United States. Table 2 summarizes the outcomes of the linear estimation. The regression for the five countries over the period 1950-2010 is reproduced in columns 2, 3 and 4 to ensure that changes in the panel composition are not driving the results. Reducing the number of countries in the sample from 11 to five does not alter the results of Table 1. The panel fixed effects regression delivers a positive coefficient (0.2). The square terms enters negatively (-5.6). The concave relationship between inequality and growth is confirmed by the piecewise regression, with the coefficient being positive (negative) below (above) the cut-off point.

However, increasing the time dimension of the panel has a strong impact on the sign of the effects. Over the period 1921-2010, the coefficient associated with the top income share is negative -0.28 and significant at 5 percent after the inclusion of the control variables in the case of a linear regression. These results are robust to the inclusion of both country and year fixed effects.[11] The negative sign is still significant at -0.3 over the period 1921 and 2000 but turns out not to be significant over the period 1931-2010 (its p-value is 15 percent).[12] The standard errors control for cross-section correlation. The results stand in contrast with the largest 11 countries linear estimation panel data that pointed to a positive sign as underlined in Andrews et al. (2011).

[11] Including a dummy for World War II does not change the results and is not shown here.
[12] See Table A4 in the appendix for the detailed regressions.

Introducing squared values of the top income share in column 6 comes out with a positive, but insignificant sign (1.6). This coefficient would have produced a convex relationship between the top income share and growth. The cut-off point corresponds to a very high level of inequality. This is in line with the results that the sign of top income share is large, negative and consistent across most specifications. These regressions indicate that the time dimension of the panel matters for the presence of non-linearity. On the post WWII data, the estimation indicates that non-linearities are important. However, over a longer time frame, the relationship appears to be linear and negative.

4 Panel threshold regression

One of the main shortcoming of the linear estimation proposed by Banerjee and Duflo (2003) is the arbitrary choice of the cut-off point in the piecewise regressions presented in Tables 1 and 2. The choice of the cut-off point is based on the estimation of the regression with a quadratic term but remains ad hoc. The panel threshold regression (PTR) has the advantage of using a statistical criteria to choose the cut-off point. The eligibility criteria is the cut-off point that minimizes the sum of squared errors (see Hansen 1999, for a presentation of PTR). A PTR consists in estimating equation 6. The dependent variable is the yearly difference of the log of GDP per capita of country i at time t. The explanatory variable is $x_{i,t-1}$ the lagged top 1 percent income share. $q_{i,t}$ is the threshold variable and $z_{i,t-1}$ is a vector of additional control variables. The PTR consists in dividing the data into two groups (two regimes), depending on whether the threshold variable $q_{i,t}$ is smaller or larger than the threshold parameter γ. The two regimes differ with respect to the slope of the effect β_1 and β_2.

$$y_{i,t} = \alpha_i + \beta_1 x_{i,t-1} I(q_{it} \leq \gamma) + \beta_2 x_{i,t-1} I(q_{it} > \gamma) + \beta_3 z_{i,t-1} + \epsilon_{i,t} \tag{6}$$

For simplicity, abstracting from $z_{i,t-1}$, equation 6 can be rewritten as:

$$y_{i,t} = \alpha_i + \beta x_{i,t-1}(\gamma) + \epsilon_{i,t} \tag{7}$$

with

$$x_{i,t-1}(\gamma) = \begin{pmatrix} x_{i,t-1} I(q_{it} \leq \gamma) \\ x_{i,t-1} I(q_{it} > \gamma) \end{pmatrix} \tag{8}$$

and $\beta = (\beta_1, \beta_2)$. After removing the averages from equation 7:

$$y^*_{i,t} = \beta x^*_{i,t-1}(\gamma) + \epsilon^*_{i,t} \tag{9}$$

with $y^*_{i,t} = y_{i,t} - \bar{y}_i$, $x^*_{i,t}(\gamma) = x_{i,t}(\gamma) - \bar{x}_i(\gamma)$, $\epsilon^*_{i,t} = \epsilon_{i,t} - \bar{\epsilon}_i$. Stacking the data over all individual countries we get:

$$Y^*_t = \beta X^*_{t-1}(\gamma) + \epsilon^*_{i,t} \tag{10}$$

The PTR consists in estimating the slope coefficient β for each of the parameter γ by ordinary least squares:

$$\hat{\beta}(\gamma) = \left(X^*(\gamma)'X^*(\gamma)\right)^{-1} X^*(\gamma)'Y^* \tag{11}$$

The vector of error is $\hat{e}^*(\gamma)$ and the sum of square residual is $S_1(\gamma)$:

$$\hat{e}^*(\gamma) = Y^* - \hat{\beta}(\gamma)X^*(\gamma)S_1(\gamma) = \hat{e}^*(\gamma)'\hat{e}^*(\gamma) \tag{12}$$

The threshold parameter $\hat{\gamma}$ is the threshold that minimizes the sum of squared residuals.

$$\hat{\gamma} = \text{argmin} \quad S_1(\gamma) \tag{13}$$

4.1 Panel 1950-2010, 11 countries

Table 3 displays the results of the PTR for the 11 countries panel over the period 1950-2010.[13] Both the exogenous variable and the threshold variable are defined by the top income share. The threshold parameter γ is comprised between -0.3039 and -0.6464 depending on the number of control variables considered. The latter corresponds to the threshold with the full set of control variables.[14] The PTR estimates a threshold much smaller than the threshold derived from the quadratic equation in Table 1. It follows that data points are distributed much more evenly around the threshold. Around 33 percent of the data points are below the threshold. This implies that for most countries data points below the threshold corresponds to years where inequality is low, i.e. the time period comprised between the early 1970s and the early 1990s as shown in Figure 3.

The sign and magnitude of the coefficient are, in addition, very different than the coefficients estimated in the piecewise regression. When top income share is below the threshold the coefficient associated with inequality is negative and comprised between -0.3 and -0.4, pointing to a negative impact of inequality on growth. When top income share is above the threshold the coefficient associated with inequality is still negative but slightly smaller as it is comprised between -0.15 and -0.2. The coefficients are relatively stable to the inclusion of control variables as shown by columns 1 to 5 in Tables A5 and A6 in the appendix. Where the piecewise regressions points to a concave relationship between inequality and growth, the PTR estimates a negative relationship with a kink. Put differently, choosing the cut-off point based on a statistical criteria does not support the concave relationship between inequality and growth. Our result indicates that when inequality is high the impact on growth is negative but not as strong as when inequality is low. In other words, inequality is especially harmful in countries that are relatively more egalitarian.

$$F_1 = \left(S_0 - S_1(\hat{\gamma})\right)/\hat{\sigma}^2 \tag{14}$$

The table also reports an F-test that determines whether the threshold effects are different across the two regimes ($\beta_1 \neq \beta_2$). The F-test in equation 14 consists in testing whether the sum of squared errors of the regression without a threshold and the sum of squared errors of the regression with a threshold are statistically different. The null hypothesis assumes that there is no threshold. The F-test rejects the null

[13] There is no need to include country fixed effects as the variables are all normalized by their average.
[14] The threshold variable is normalized by the average. The Tables A5 and A6 in appendix presents the regressions including control variables one at a time.

Table 3: Panel threshold regression - 1950-2010 - 11 countries

	(1)	(2)	(3)	(4)
ID	11	11	11	11
	1950-2010	1950-2010	1950-2010	1950-2010
	single	single	double	double
$Top1_{t-1}$<Thr1	-0.346***	-0.150*	-0.542***	-0.141*
	(0.092)	(0.081)	(0.116)	(0.081)
$Top1_{t-1}$>Thr1	-0.225***	-0.002	-0.416***	-0.872***
	(0.064)	(0.060)	(0.094)	(0.243)
$Top1_{t-1}$>Thr2			-0.327***	-0.004
			(0.073)	(0.059)
$lgdppk_{t-1}$	-0.015***	-0.041***	-0.015***	-0.040***
	(0.005)	(0.006)	(0.005)	(0.006)
hc_{t-1}	0.002	-0.021**	0.001	-0.020**
	(0.008)	(0.009)	(0.008)	(0.008)
$\left(\frac{i}{y}\right)_{t-1}$	-0.063***	-0.098***	-0.063***	-0.099***
	(0.023)	(0.022)	(0.022)	(0.022)
π_{t-1}	-0.030***	-0.014**	-0.028***	-0.014**
	(0.005)	(0.006)	(0.005)	(0.006)
_cons	0.222***	0.465***	0.237***	0.463***
	(0.028)	(0.050)	(0.029)	(0.049)
Threshold1	-0.6464	-0.6732	-0.6464	-0.6775
Threshold2			0.3280	-0.6778
F-stat	9.51*	19.69***	7.52	8.27
Num of id<Thr1	218	207	218	205
Num of id<Thr2			433	202
N	660	660	660	660
R^2	0.192	0.508	0.201	0.516
$YearFE$	No	Yes	No	Yes

Standard errors in parentheses * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$ This table summarizes the results of the threshold regression for a panel of 11 countries over the period 1950-2010. The dependent variable is the difference of the log of GDP per capita in 1990 international GK $. The exogenous variable is the lagged top 1 percent income share. The control variables are the lagged GDP per capita in log, lagged human capital, lagged investment to GDP ratio, and lagged inflation rate. The threshold is the standardized top income share. The 11 countries are: Australia, Canada, Denmark, Finland, France, Japan, New Zealand, Norway, Sweden, the United Kingdom, and the United States.

Figure 3: Distribution of data points across regimes $\gamma = -0.6464$

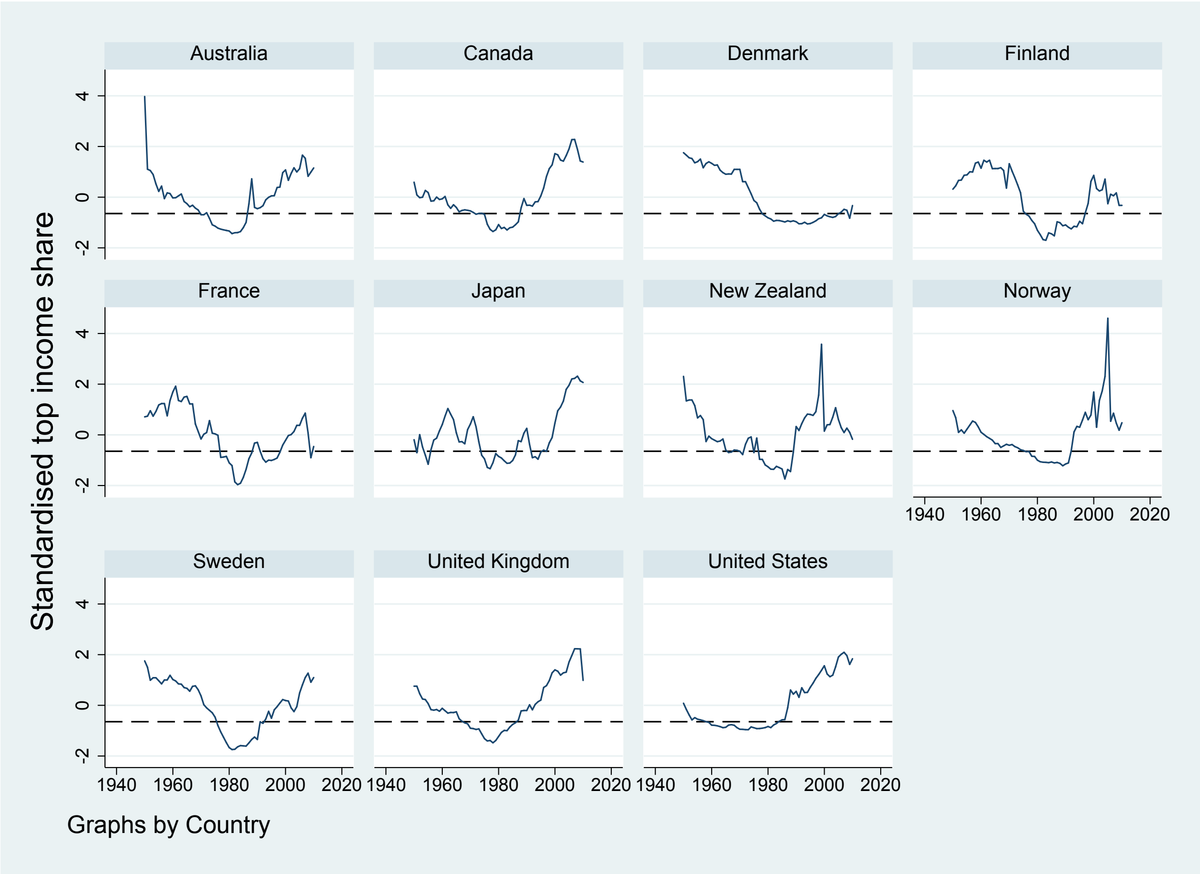
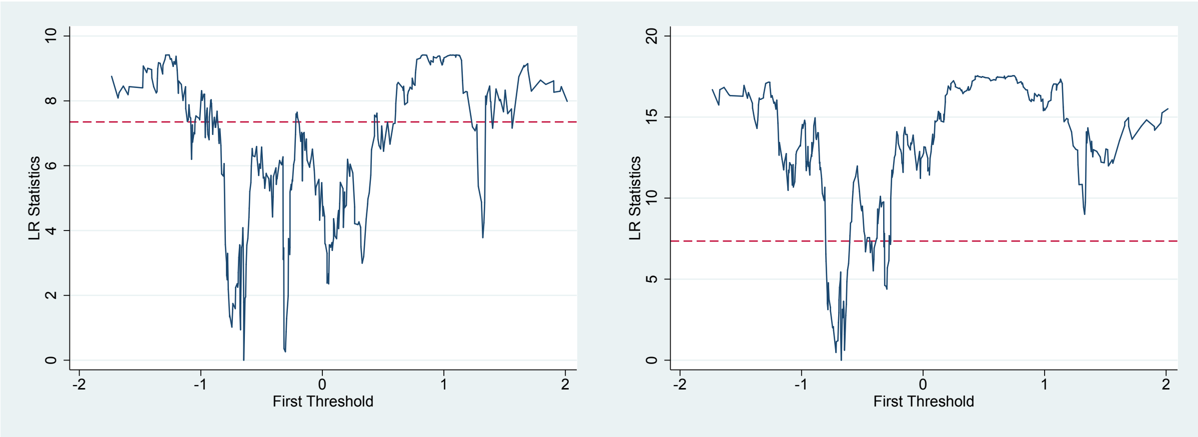
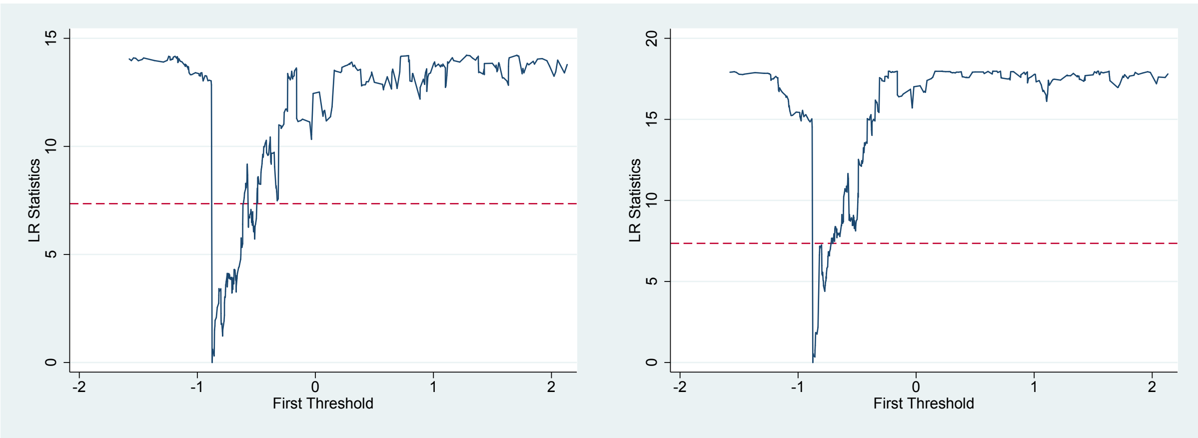


Figure 4: Likelihood ratio



(a) 11 countries

(b) 11 countries - time fixed effect



(c) 5 countries

(d) 5 countries - time fixed effect

hypothesis of no threshold. The result of this test concludes in favor of a non-linear relationship between inequality and growth against a linear relationship.

Adding time fixed effects in regression 2 in Table 3 modifies the results to the extent that the parameter β_1 is now smaller and comprised between -0.22 and -0.15. The later corresponds to the regression with all the control variables.[15] Additionally, the high inequality regime is associated with a parameter β_2, which is not statistically different from zero. The F-test in Table 3 rejects the null hypothesis that the threshold effects are similar across regimes $\beta_1 \neq \beta_2$. The evidence of a non-linear relationship is robust to the inclusion of time fixed effects. While, inequality negatively impacts growth in the low inequality regime regardless of the inclusion of time fixed effects, the sign associated with high inequality is either negative or zero depending on the inclusion of time fixed effects. One question that arises is whether a refinement of the estimation procedure could help us identify whether β_2 is negative, zero or positive. This is the purpose of the next section, which uses panel smooth transition regression.

[15] The threshold variable is normalized by the average. The Tables A7 and A8 in the appendix presents the regressions including control variables one at a time.

While equation 14 tests whether $\beta_1 = \beta_2$, the likelihood ratio LR_1 in equation 15 tests whether the threshold parameter is the true threshold parameter $H_0 : \hat{\gamma} = \gamma_0$. This test helps us to form a no-rejection region:

$$LR_1 = (S_1(\gamma) - S_1(\hat{\gamma})) / \hat{\sigma}^2 \tag{15}$$

In Figure 4a, the red horizontal dotted line represents the critical value at 5 percent. Values of LR_1 greater than 7.35 reject the null hypothesis. The test is performed with all the control variables corresponding to regression 1 in Table 3. The figure shows that in addition to our threshold -0.6464, there is the possibility of alternative thresholds.

Estimating the regression for two thresholds confirm the existence of a second threshold at 0.3280. This second threshold corresponds to a level of high inequality and around one third of the data points are above this threshold. The coefficients are negative across the three regimes and declining in size as inequality is rising: $\beta_1 = -0.542$, $\beta_2 = -0.416$ and $\beta_3 = -0.327$. In the presence of a double threshold, the F-test compares the sum of square error of the model with one threshold with the sum of square error of the model with two thresholds. The p-value associated with the F-test exceeds 10 percent.

Estimating a model with two thresholds and time fixed effects produces weak results. The two thresholds are very similar to each others -0.6775 and -0.6778. The F-test tends to conclude that there is no difference between the threshold 1 and the threshold 2. The likelihood ratio in Figure 4 tends to reject the uniqueness of the threshold parameter.

4.2 Panel 1920-2010, five countries

The results of the PTR for the panel with a longer time dimension (1920-2010) for a smaller subset of countries is reproduced in Table 4. As a first step, regressions 1 and 2 estimate the threshold over the period 1950-2010 in order to understand how the composition of the panel impacts the results. In regression 1, the results are in line with the results based on the 11 countries panel. The positive coefficient in the linear fixed effects panel regression becomes negative in the threshold regression. In the low inequality regime the coefficient is -0.387 and the coefficient is -0.173 in the high inequality regime. The evidence of a concave relationship that comes up when including a square term is not supported when the cut-off point is chosen based on a statistical criteria. Interestingly, including time fixed effects has a strong impact on the coefficients as they turn insignificant. The value of the coefficient will be further discussed in the section in the light of the panel smooth transition regression.

Over the period 1920-2010, the results are supportive of a non-linear relationship where the coefficients are negative in both regimes but the effect is smaller in the high inequality regime. In addition, the results are robust to the inclusion of time fixed effects. 15 percent of the data points are below the threshold, which has a value of -0.87. The time period for which data points are below the threshold includes mostly the 1970s and the 1980s in line with the 11 country panel. For the United States, the observations from 1953 to 1985 are below the threshold. In the regime with low inequality, the top income share has a negative impact on growth with a parameter β_1 ranging from -0.9 to -0.67, the former corresponding to the regression with the full set of control variables.[16] In the regime with high inequality, the top income share also has a negative impact on growth with a smaller coefficient fluctuating between -0.35 and -0.25.

[16] Tables A7 and A8 in the appendix present the regressions including control variables one at a time.

Table 4: Panel threshold regression - 1921-2010 - five countries

	(1)	(2)	(3)	(4)	(5)	(6)
ID	5	5	5	5	5	5
$year$	1950-2010	1950-2010	1921-2010	1921-2010	1921-2010	1921-2010
	single	single	single	single	double	double
$Top1_{t-1}$<Thr1	-0.387***	-0.014	-0.870***	-0.722***	-0.387*	-0.722***
	(0.139)	(0.102)	(0.199)	(0.264)	(0.208)	(0.264)
$Top1_{t-1}$>Thr1	-0.173*	0.096	-0.347***	-0.404**		
	(0.089)	(0.080)	(0.114)	(0.176)		
$Top1_{t-1}$>Thr2					-0.218*	-0.404**
					(0.116)	(0.176)
$lgdppk_{t-1}$	-0.031***	-0.065***	-0.021***	-0.044***	-0.018***	-0.044***
	(0.003)	(0.006)	(0.006)	(0.015)	(0.006)	(0.015)
$\left(\frac{i}{y}\right)_{t-1}$	0.079*	0.101**	0.195***	0.065	0.171***	0.065
	(0.043)	(0.048)	(0.064)	(0.091)	(0.065)	(0.091)
π_{t-1}	-0.145***	-0.188***	-0.005	-0.014	-0.003	-0.014
	(0.034)	(0.041)	(0.024)	(0.031)	(0.024)	(0.031)
_cons	0.338***	0.596***	0.217***	0.479***	0.177***	0.479***
	(0.032)	(0.054)	(0.055)	(0.133)	(0.056)	(0.133)
Threshold1	-0.0478	-0.6462	-0.8746	-0.8750	-0.8750	-0.8750
Threshold2					-0.8806	-0.8806
F-stat	11.91**	9.97	14.44 *	24.35 **	174.73***	172.17***
Num of id<Thr1	251	137	66	66	65	65
Num of id<Thr2					1	1
N	305	305	445	445	445	445
R^2	0.299	0.688	0.068	0.311	0.034	0.311
$YearFE$	no	yes	no	yes	no	yes

Robust Standard errors in parentheses * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$ This table summarizes the results of the panel threshold regression over 1921-2010 for five countries: Australia, Canada, France, Japan and the United States. The dependent variable is the difference of the log of GDP per capita in 1990 international GK $. The exogenous variable is the lagged top 1 percent income share. The control variables are the lagged GDP per capita in log, lagged investment to GDP ratio, and lagged inflation rate. The threshold is the standardized top income share normalized by the average and the standard deviation.

The F-test rejects the null hypothesis of no threshold. The likelihood ratio in Figure 4c tends to rule out the possibility of a second threshold. In fact, the estimation allowing for two thresholds in column 5 excludes this possibility. Comparing the linear regression versus the threshold regression, the PTR tends to deliver a stronger negative effect whereas the linear regression produces a coefficient similar to the high inequality regime. In addition, the PTR produces noticeably larger effects for the five country panel rather than for the 11 country panel databases regardless of the regime considered. Lastly, the inclusion of a dummy variable for World War II does not alter the results and thus is not shown here.

Including time fixed effects in the regression impacts the results marginally (regression 2 in Table 4). In line with the previous section, the coefficients for the low inequality regime are slightly reduced. However, the coefficients for the high inequality regime are increased by the inclusion of the time fixed effects. It follows that the longer time frame delivers negative coefficients for both regimes regardless of the specification considered. The F-test rejects the null hypothesis of no threshold and the two thresholds regression is not conclusive.

The take-home message from these PTR estimations is that there is not only evidence of a non-linear relationship between inequality and growth but that the threshold regressions have a large impact on the sign of the coefficient. While the low inequality regime is associated with a negative sign regardless of the panel composition (11 countries versus five countries) or the time frame considered (1950-2010 versus 1920-2010), the sign of the high inequality regime is only robust to the inclusion of time fixed effects when considering the long panel (1920-2010). In the next section, we use panel smooth transition regression to refine the estimation of the high inequality regime over the period 1950-2010.

5 Panel smooth transition regression

Regarding the regression over the period 1950-2010, the coefficient associated with the high inequality threshold is negative but insignificant when time fixed effects are included in the Tables 3 and 4. One question that arises is whether alternative estimation techniques could give more precise information about the value of this coefficient and about its sign (negative or positive). In this section we use panel smooth transition regression (PSTR) to refine the estimation of the coefficients (see González et al. 2004; Colletaz and Hurlin 2006).

The shortcoming of the panel threshold regression presented in the previous section is that it allows for a small number of regimes. In addition, the transition from one regime to another regime is dichotomic. These shortcomings are overtaken by panel smooth transition regression which allows for a continuum of regimes. Eq 16 illustrates the PSTR with q_{it} the threshold variable. The transition function $h(q_{it};\gamma,c_z)$ is a continuous and bounded function of q_{it} with γ the slope of the transition function and c_z the location parameter.

$$y_{i,t} = \alpha_i + \beta_1 x_{i,t-1} + \beta_2 x_{i,t-1} h(q_{it};\gamma,c_z) + \beta_3 z_{i,t-1} + \epsilon_{i,t} \tag{16}$$

The transition function is expressed as a logistic function:

$$h(q_{it};\gamma,c_z) = \left[1 + e^{-\gamma \prod_{z=1}^{m}(q_{it}-c_z)}\right]^{-1} \tag{17}$$

When the threshold variable and the explanatory variable are different, the elasticity can be expressed as follow:

$$\frac{\partial y_{i,t}}{\partial x_{i,t-1}} = \beta_1 + \beta_2 h(q_{it}; \gamma, c_z) \tag{18}$$

In its general form, the PSTR allows for a number r of transition functions:

$$y_{i,t} = \alpha_i + \beta_1 x_{i,t-1} + \sum_{j=1}^{r} \beta_j x_{i,t-1} h_j(q_{it}^j; \gamma_j, c_{j,z}) + \beta_3 z_{i,t-1} + \epsilon_{i,t} \tag{19}$$

with

$$h_j(q_{it}^j; \gamma_j, c_{j,z}) = \left[1 + e^{-\gamma_j \prod_{z=1}^{m_j} (q_{it}^j - c_{j,z})}\right]^{-1} \tag{20}$$

The estimation of the PSTR requires to choose the number of transition functions, r. In a first step, choosing the number of transition functions is equivalent to testing for linearity against the PSTR. Following González et al. (2004), testing whether $\beta_2 = 0$ in equation 16 is equivalent to taking a first-order Taylor expansion of equation 16 and testing whether $\beta_2^* = ... = \beta_m^* = 0$ in equation 21.

$$y_{i,t} = \alpha_i + \beta_1^* x_{i,t-1} + \beta_2^* x_{i,t-1} q_{it} + ... + \beta_m^* x_{i,t-1} q_{it}^m + \beta_3 z_{i,t-1} + \epsilon_{i,t} \tag{21}$$

The test compares the residual sum of square SSR_0 of equation 21 and the residual sum of square of equation 16.

$$F = \frac{(SSR_0 - SSR_1)}{mk} \frac{(TN - N - mk)}{SSR_0} \tag{22}$$

with k the number of explanatory variables. Following the same logic, one can perform a similar test to discriminate between the model with one transition function and the model with two transition functions. The tests are illustrated in Table 5 for the model with and without time fixed effects. The null hypothesis of no transition function can be rejected irrespective of the number of location parameters for the model without time fixed effects. This is indicative of the presence of non-linearities. When time fixed effects are included, the null hypothesis is rejected when $m = 2$. In addition, the tests reject the model with 2 transition functions irrespective of the time fixed effects.

Table 6 presents the results of the PSTR for both panels with and without time fixed effects. The results are in line with the PTR. When the top income share is low, the impact on growth of an increase in inequality is negative. The parameter $\hat{\beta}_1$ is comprised between -0.72 and -0.4 depending on the type of control variables specified and depending on the inclusion of time fixed effects for the 11 country panel. The regressions including the control variables one at a time are presented in Tables A9 and A10 in the appendix. Interestingly, the coefficient associated with the low inequality regime is larger than in the PTR (comprised between -0.6 and -0.2). In the PSTR, the coefficient $\hat{\beta}_2$ is positive and comprised between 0.2 and 0.4 indicating that when inequality grows, the size of the effect diminishes. This result is also in line with the PTR as the coefficient associated with the high inequalities regime was either negative (comprised between -0.4 and -0.2) but smaller than the coefficient associated with the low inequality regime or not statistically different from zero. Interestingly, the coefficient associated with high inequality is statistically different from zero. This validates our methodological choice to rely on

Table 5: Tests for remaining non-linearity

id	11 countries				5 countries			
year	1950-2010				1950-2010			
TimeFE	no		yes		no		yes	
Locationparameters	$m=1$	$m=2$	$m=1$	$m=2$	$m=1$	$m=2$	$m=1$	$m=2$
$H_0: r=0 \quad vs \quad H_1: r=1$	2.99	3.83	0.84	7.27	4.89	4.83	2.27	11.36
	(0.08)	(0.02)	(0.35)	(0.00)	(0.03)	(0.01)	(0.13)	(0.00)
$H_0: r=1 \quad vs \quad H_1: r=2$	0.21	1.16	-	1.64	0.04	0.06	-	8.38
	(0.64)	(0.30)	-	(0.19)	(0.84)	(0.93)	-	(0.00)

This table summarizes the F-test of remaining non-linearity. The first line tests a linear model ($r = 0$) against a model with one threshold ($r = 1$). If the null hypothesis is rejected, a new test is performed between a single transition function against two transition functions. r is the number of transition function and m is the number of location parameters. P-values are in parentheses.

PSTR to estimate with more precision the high inequality regime coefficient. The slope of the transition function γ is moderate and comprised between 2.5 and 3. A large value of γ (greater than 10 for instance) would indicate that the transition between the two regimes is similar to a PTR.

Similar results hold for the five country panel over the period 1950-2010. The coefficient associated with low inequality β_1 is -0.8 far greater than the coefficient obtained from the PTR (-0.38). The coefficient associated with high inequality is positive and significant ($\beta_2 = 0.45$) indicating that the effect decreases in size from the low to the high-inequality regime. In contrast with the PTR, the coefficients are robust to the inclusion of time fixed effects in the PSTR with $\beta_1 = -0.97$ and $\beta_2 = 0.7$. The change in the country composition of the panel has a limited impact on the slope of the transition function with γ being either 3.3 or 2.2.

The first step in the PSTR is to remove the country fixed effects by centering the variables on their individual means. It follows that $x_{i,t-1}$ is the top income share normalized by its average in equation 16. In addition, the threshold variable is the top income share normalized by its average and its standard error $q_{it} = \frac{x_{i,t}}{sd(x_i)}$. This standardization has an impact on the elasticities. On the one hand, we could assume that $\frac{\partial x_{i,t}}{\partial x_{i,t-1}} = 0$ as we do not estimate a system of equation. Under this assumption, the elasticity follows the definition in equation 18. On the other hand, top income share displays strong auto-correlation. In the latter case, the elasticity of growth to top income share would read as follows, ignoring the difference between $x_{i,t}$ and $x_{i,t-1}$:

$$\frac{\partial y_{i,t}}{\partial x_{i,t-1}} = \beta_1 + \beta_2 h(q_{it}; \gamma, c_z) + \beta_2 x_{it-1} \frac{\partial h(q_{it}; \gamma, c_z)}{\partial x_{i,t}} \tag{23}$$

In turn, the first derivative of the transition function is equal to:

$$\frac{\partial h(q_{it}; \gamma, c_z)}{\partial x_{i,t}} = -\frac{\frac{-\gamma}{sd(x_i)} e^{-\gamma(q_{it}-c_z)}}{\left(1 + e^{-\gamma(q_{it}-c_z)}\right)^2} \tag{24}$$

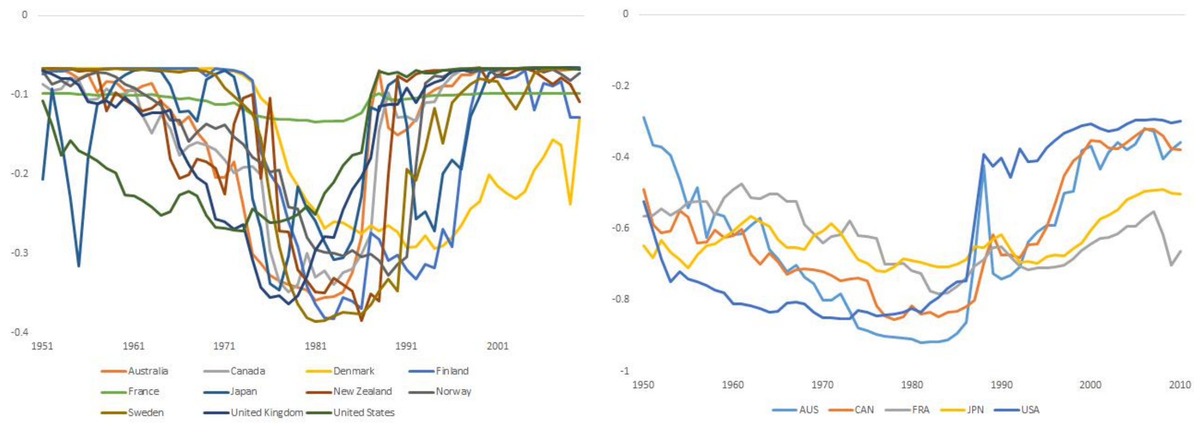
Figure 5 displays the conservative elasticity as defined in equation 18. The signs of the elasticities are not affected by the definition chosen. The impact of top income share is always negative for each of the

Table 6: Panel smooth transition regression

	(1)	(2)	(3)	(4)
ID	11 countries	11 countries	5 countries	5 countries
	1950-2010	1950-2010	1950-2010	1950-2010
(m, r^*)	(1,1)	(1,1)	(1,1)	(1,1)
β_1	-0.564***	-0.405***	-0.792***	-0.979***
	(0.145)	(0.124)	(0.225)	(0.285)
β_2	0.253***	0.339***	0.446***	0.690***
	(0.075)	(0.075)	(0.116)	(0.164)
$lgdppk_{t-1}$	-0.015**	-0.040***	-0.031***	0.043***
	(0.005)	0.005)	(0.002)	(0.004)
hc_{t-1}	-0.001	-0.021**		
	(0.008)	0.008)		
$\left(\frac{i}{y}\right)_{t-1}$	-0.062***	-0.101***	0.088**	0.032
	(0.022)	(0.022)	(0.042)	(0.048)
π_{t-1}	-0.0003***	-0.0001**	-0.138***	-0.191***
	(0.00)	(0.00)	(0.032)	(0.042)
γ	2.46	3.00	3.29	2.20
c	-0.49	-0.82	-0.11	-0.56
RSS	0.33	0.19	0.12	0.08
AIC	-7.58	7.90	-7.72	-7.95
N	660	660	305	305
R^2	0.193	0.509	0.29	0.51
$YearFE$	No	Yes	No	Yes

Standard errors in parentheses * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$ This table summarizes the results of the panel smooth transition regression for both panel over the period 1950-2010. The dependent variable is the difference of the log of GDP per capita in 1990 international GK $. The exogenous variable is the lagged top 1 percent income share. The control variables are the lagged GDP per capita in log, lagged human capital, lagged investment to GDP ratio, and lagged inflation rate. The threshold is the standardized top income share. γ and c are the parameters of the logistic function.

Figure 5: Elasticity of growth to top income share, 1950-2010



(a) 11 countries

(b) Five countries

countries of the panel. The elasticity fluctuates between -0.4 and -0.06. The evolution of the coefficient over time reflects the evolution of the top income share. The U-shaped form of the top income share over time with a historical low in the 1970s is translated into a similarly shaped evolution of the elasticities. Against this backdrop, at the country level, elasticities display heterogenous pattern with respect to the timing of the decline and the timing of the recovery. For instance, the elasticity in the United States drops in the 1950s but has already recovered in the early 1990s. On the contrary, the elasticity in Denmark is constant until the 1970s and has not yet recovered from its historical low in 1990. Countries also differ with respect to the amplitude of the elasticity over time. In France a minimum is reached at -0.13, while in Sweden or Finland the minimum is 3 times that of France at -0.38.

The right-hand side graph confirms these results looking at the panel with five countries over the period 1950-2010. The elasticities are always negative and fluctuate between -0.2 and -1. The elasticities reach a low point between the late 1970s and early 1980s. The increase in the elasticities is more pronounced in the countries that experienced a marked increase in inequality such as the United-States, Australia and Canada. In contrast, the elasticities are much more stable throughout the period in France and Japan.

6 Conclusion

In this paper we have revisited the empirical literature on inequality and growth by taking advantage of the long time dimension offered by the availability of the top income database. The long time dimension of the data has enabled us to test whether the relationship between inequality and growth is linear or non-linear for different time periods and different compositions of the panel.

We show that the relationship is best described by non-linearity whereas the quasi totality of existing papers have focused on linear estimation. Additionally, adopting a non-linear approach has important consequence for the sign of the effects. Fixed effects panel regressions find a positive effect between inequality and growth. Contrastingly, PTR and PSTR estimate a negative relationship between inequality and growth. In particular, the effect is larger in the low inequality regime rather than in the high inequality regime. This result also stands in contrast with Banerjee and Duflo (2003) who point toward a concave relationship. The advantage of our approach is that the threshold is chosen according to a statistical criteria, while the choice of the cut-off point is somewhat adhoc in Banerjee and Duflo (2003). Lastly, individual country elasticities indicate that the size of the effect is heterogenous across countries. As the threshold is function of top income share, differences exist between countries with a L-shaped pattern and countries with a U-shaped pattern of inequality.

The negative and convex relationship between inequality and growth is less intuitive than the concave relationship highlighted by the piecewise regression. A possible interpretation is that inequality is more harmful in homogenous societies where inequality levels are low. An increase in inequality in an egalitarian society may bring more negative consequences than if the same increase takes place in an already unequal economy. Put differently, the marginal effect of an increase in inequality is decreasing with the inequality level.

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Appendix

Existing results of Herzer and Vollmer (2013) using an error correction model

Another approach developed by Herzer and Vollmer (2013) is to rely on cointegration to measure the impact of top income share on growth in the long run. Their main result is that top income share has a negative impact on growth for a panel of 11 countries over the period 1960-1996. The coefficient is large and equal to -0.93. This result is robust to the inclusion of earlier data point as the coefficient is -1.16 over the period 1950-1996. However the coefficient is not statistically different from zero when adding later data point as for instance when the regression is performed over the period 1961-2010 or over the entire sample 1950-2010.

Table A1: Reproducing the results of Herzer and Vollmer(2013)

	(1)	(2)	(3)	(4)	(5)	(6)
	1961-1996	1950-1996	1961-2010	1950-2010	w/oFRN	w/oJPN
Log(Top1)	-0.937***	-1.165***	0.083	-0.056	-0.084	-0.134
	(0.177)	(0.399)	(0.145)	(0.229)	(0.246)	(0.095)
Log(open)	0.506**	-0.378	1.059***	1.070***	1.031**	1.110**
	(0.190)	(0.250)	(0.301)	(0.414)	(0.456)	(0.438)

Standard errors in parentheses * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. The estimation is performed using panel dynamic OLS with group-mean using 11 countries. The DOLS is estimated with lag=lead=1. The dependent variable is the log of GDP per capita. The explanatory variable is the log of top 1 percent income share and the control variable is the log of the openness indicator defined as export plus import over GDP.

Additional regressions

Table A2: Panel fixed effects - 1950-2010 - 11 countries

	(1) 1960-2000	(2) 1950-2000	(3) 1960-2010	(4) 1950-2010	(5) w/oFRN	(6) w/oJPN	(7) No extractive
L.Top1	0.177**	0.171**	0.111	0.100*	0.106	0.076	0.084
	(0.071)	(0.074)	(0.062)	(0.052)	(0.068)	(0.046)	(0.061)
L.lgdppk	-0.064***	-0.036***	-0.069***	-0.042***	-0.040***	-0.046***	-0.039***
	(0.008)	(0.006)	(0.008)	(0.004)	(0.004)	(0.010)	(0.005)
L.hc	-0.020**	-0.022**	-0.020***	-0.020***	-0.025**	-0.021***	-0.017**
	(0.009)	(0.008)	(0.005)	(0.005)	(0.010)	(0.005)	(0.005)
L.csh_i	-0.135***	-0.112***	-0.109***	-0.091***	-0.100***	-0.108**	-0.093*
	(0.038)	(0.034)	(0.028)	(0.027)	(0.027)	(0.036)	(0.040)
L.PI	-0.015	-0.012	-0.015	-0.012	-0.014*	-0.011	-0.023**
	(0.009)	(0.007)	(0.010)	(0.007)	(0.008)	(0.007)	(0.009)
_cons	0.692***	0.413***	0.737***	0.462***	0.455***	0.504***	0.432***
	(0.084)	(0.059)	(0.073)	(0.039)	(0.041)	(0.103)	(0.038)
N	451	550	561	660	600	600	480
R^2	0.496	0.416	0.563	0.493	0.487	0.466	0.542
ID	11	11	11	11	10	10	8
$CountryFE$	Yes	Yes	Yes	Yes	Yes	Yes	Yes
$YearFE$	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Standard errors in parentheses * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. This table summarizes the results of the country and time fixed effects regressions for a panel of 11 countries over the period 1950-2010. The dependent variable is the difference of the log of GDP per capita in 1990 international GK $. The exogenous variable is the lagged top 1 percent income share. The control variables are the lagged GDP per capita in log, lagged human capital, lagged investment to GDP ratio, and lagged inflation rate. The 11 countries are: Australia, Canada, Denmark, Finland, France, Japan, New Zealand, Norway, Sweden, the United Kingdom, and the United States. Column 7 excludes extractive countries: Australia, Canada and Norway.

Table A3: Top income share squared and piecewise regressions

	(1) squared	(2) piecewise 1	(3) piecewise 2	(4) piecewise 3	(5) piecewise 4	(6) piecewise 5
$Top1_{t-1}$	0.217***					
	(0.063)					
$Top1^2_{t-1}$	-4.810***					
	(1.464)					
$Top1_{t-1} < \gamma$		0.203**	0.252***	0.246***	0.313***	0.272***
		(0.081)	(0.073)	(0.066)	(0.060)	(0.073)
$Top1_{t-1} > \gamma$		-0.287*	-0.308**	-0.304**	-0.309**	-0.314**
		(0.159)	(0.122)	(0.125)	(0.112)	(0.114)
$lgdppk_{t-1}$	-0.043***		-0.054***	-0.053***	-0.046***	-0.043***
	(0.004)		(0.005)	(0.006)	(0.004)	(0.004)
hc_{t-1}	-0.020***			-0.008	-0.020***	-0.020***
	(0.005)			(0.007)	(0.005)	(0.005)
$\left(\frac{i}{y}\right)_{t-1}$	-0.099***				-0.092***	-0.097***
	(0.027)				(0.028)	(0.028)
π_{t-1}	-0.000					-0.000
	(0.000)					(0.000)
_cons	0.476***	0.029*	0.493***	0.506***	0.499***	0.479***
	(0.043)	(0.014)	(0.049)	(0.061)	(0.042)	(0.043)
N	660	660	660	660	660	660
R^2	0.501	0.400	0.484	0.485	0.500	0.503
$CountryFE$	YES	YES	YES	YES	YES	YES
$TimeFE$	YES	YES	YES	YES	YES	YES

Standard errors in parentheses * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. This table summarizes the results of the regressions with squared inequality and piecewise regressions for a panel of 11 countries over the period 1950-2010. The dependent variable is the difference of the log of GDP per capita in 1990 international GK $. The exogenous variable is the lagged top 1 percent income share. The control variables are the lagged GDP per capita in log, lagged human capital, lagged investment to GDP ratio, and lagged inflation rate. The 11 countries are: Australia, Canada, Denmark, Finland, France, Japan, New Zealand, Norway, Sweden, the United Kingdom, and the United States. Column 7 excludes extractive countries: Australia, Canada and Norway.

Table A4: Linear regression top income share and growth: 1921-2010; 5 countries

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	1921-2010	1921-2010	1921-2010	1921-2010	1921-2000	1931-2010	squared
$Top1_{t-1}$	-0.168	-0.245	-0.260***	-0.280**	-0.352*	-0.327	-0.381
	(0.124)	(0.144)	(0.056)	(0.094)	(0.140)	(0.187)	(0.197)
$lgdppk_{t-1}$		-0.036**	-0.037	-0.041	-0.041	-0.041	-0.046
		(0.012)	(0.018)	(0.029)	(0.039)	(0.033)	(0.030)
$\left(\frac{i}{y}\right)_{t-1}$			0.059	0.077	0.074	0.059	0.104
			(0.415)	(0.453)	(0.518)	(0.507)	(0.467)
π_{t-1}				-0.021	-0.024	-0.015	-0.030
				(0.071)	(0.077)	(0.081)	(0.077)
$Top1^2_{t-1}$							4.201
							(4.951)
_cons	-0.010	0.292***	0.292***	0.332	0.345	0.296	0.359*
	(0.062)	(0.045)	(0.041)	(0.159)	(0.225)	(0.214)	(0.158)
N	449	449	449	449	394	400	495
R^2	0.288	0.302	0.303	0.304	0.295	0.286	0.358
ID	5	5	5	5	5	5	5
$CountryFE$	Yes	Yes	Yes	Yes	Yes	Yes	Yes
$YearFE$	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Robust standard errors in parentheses * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$ This table summarizes the results of the panel regression over the period 1921-2010 for five countries: Australia, Canada, France, Japan and the United States. The dependent variable is the difference of the log of GDP per capita in 1990 international GK $. The exogenous variable is the lagged top 1 percent income share. The control variables are the lagged GDP per capita in log, lagged investment to GDP ratio, and lagged inflation rate.

Table A5: Panel threshold regression - 1950-2010 - 11 countries

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Single1	Single2	Single3	Single4	Single5	Double	No extractive
$Top1_{t-1}$<Thr1	-0.407***	-0.317***	-0.312***	-0.299***	-0.346***	-0.542***	-0.442***
	(0.097)	(0.096)	(0.096)	(0.095)	(0.092)	(0.116)	(0.107)
$Top1_{t-1}$>Thr1	-0.200***	-0.160**	-0.159**	-0.156**	-0.225***	-0.416***	-0.281***
	(0.066)	(0.066)	(0.066)	(0.066)	(0.064)	(0.094)	(0.073)
$Top1_{t-1}$>Thr2						-0.327***	
						(0.073)	
$lgdppk_{t-1}$		-0.019***	-0.025***	-0.020***	-0.015***	-0.015***	-0.015***
		(0.002)	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)
hc_{t-1}			0.012	0.001	0.002	0.001	0.003
			(0.008)	(0.008)	(0.008)	(0.008)	(0.009)
$\left(\frac{i}{y}\right)_{t-1}$				-0.061***	-0.063***	-0.063***	-0.032
				(0.023)	(0.023)	(0.022)	(0.023)
π_{t-1}					-0.030***	-0.028***	-0.034***
					(0.005)	(0.005)	(0.007)
_cons	0.044***	0.217***	0.250***	0.242***	0.222***	0.237***	0.230***
	(0.006)	(0.020)	(0.029)	(0.029)	(0.028)	(0.029)	(0.030)
Threshold1	-0.6464	-0.3039	-0.3039	-0.3039	-0.6464	-0.6464	-0.3070
Threshold2						0.3280	
F-stat	24.61*	16.38**	15.63**	13.72**	9.51*	7.52	12.15*
Num of id<Thr1	218	291	291	291	218	218	215
Num of id<Thr2						433	
N	660	660	660	660	660	660	480
R^2	0.038	0.145	0.148	0.157	0.192	0.201	0.257
ID	11	11	11	11	11	11	8
$CountryFE$	Yes	Yes	Yes	Yes	Yes	Yes	Yes
$YearFE$	No	No	No	No	No	No	No

Standard errors in parentheses * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$ This table summarizes the results of the threshold regression for a panel of 11 countries over the period 1950-2010. The dependent variable is the difference of the log of GDP per capita in 1990 international GK $. The exogenous variable is the lagged top 1 percent income share. The control variables are the lagged GDP per capita in log, lagged human capital, lagged investment to GDP ratio, and lagged inflation rate. The threshold is the standardized top income share. The 11 countries are: Australia, Canada, Denmark, Finland, France, Japan, New Zealand, Norway, Sweden, the United Kingdom, and the United States.

Table A6: Panel threshold regression - 1950-2010 - 11 countries - time fixed effects

	(1) Single1	(2) Single2	(3) Single3	(4) Single4	(5) Single5	(6) Double	(7) No extractive
$Top1_{t-1}$<Thr1	-0.222**	-0.166**	-0.171**	-0.134*	-0.150*	-0.141*	-0.212***
	(0.086)	(0.081)	(0.081)	(0.080)	(0.081)	(0.081)	(0.082)
$Top1_{t-1}$>Thr1	-0.054	-0.014	-0.018	0.024	-0.002	-0.872***	-0.032
	(0.061)	(0.057)	(0.057)	(0.058)	(0.060)	(0.243)	(0.058)
$Top1_{t-1}$>Thr2						-0.004	
						(0.059)	
$lgdppk_{t-1}$		-0.052***	-0.051***	-0.045***	-0.041***	-0.040***	-0.040***
		(0.005)	(0.005)	(0.006)	(0.006)	(0.006)	(0.006)
hc_{t-1}			-0.010	-0.021**	-0.021**	-0.020**	-0.020**
			(0.008)	(0.009)	(0.009)	(0.008)	(0.009)
$\left(\frac{i}{y}\right)_{t-1}$				-0.088***	-0.098***	-0.099***	-0.097***
				(0.022)	(0.022)	(0.022)	(0.022)
π_{t-1}					-0.014**	-0.014**	-0.020***
					(0.006)	(0.006)	(0.007)
_cons	0.036***	0.482***	0.498***	0.487***	0.465***	0.463***	0.477***
	(0.009)	(0.048)	(0.049)	(0.049)	(0.050)	(0.049)	(0.054)
Threshold1	-0.6464	-0.6464	-0.6464	-0.6464	-0.6732	-0.6775	-0.7262
Threshold2						-0.6778	
F-stat	18.08*	16.87**	17.01**	18.51**	19.69***	8.27	27.94**
Num of id<Thr1	218	218	218	218	207	205	151
Num of id<Thr2						202	
N	660	660	660	660	660	660	480
R^2	0.412	0.491	0.492	0.506	0.508	0.516	0.624
$CountryFE$	Yes	Yes	Yes	Yes	Yes	Yes	Yes
$YearFE$	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Standard errors in parentheses * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$ This table summarizes the results of the threshold regression for a panel of 11 countries over the period 1950-2010. The dependent variable is the difference of the log of GDP per capita in 1990 international GK \$. The exogenous variable is the lagged top 1 percent income share. The control variables are the lagged GDP per capita in log, lagged human capital, lagged investment to GDP ratio, and lagged inflation rate. The threshold is the standardized top income share. The 11 countries are: Australia, Canada, Denmark, Finland, France, Japan, New Zealand, Norway, Sweden, the United Kingdom, and the United States.

Table A7: Panel threshold regression - 1921-2010 - 5 countries

	(1) Single1	(2) Single2	(3) Single3	(4) Single4	(5) Double5
$Top1_{t-1}$<Thr1	-0.677***	-0.899***	-0.864***	-0.870***	-0.387*
	(0.184)	(0.198)	(0.196)	(0.199)	(0.208)
$Top1_{t-1}$>Thr1	-0.254***	-0.423***	-0.341***	-0.347***	
	(0.091)	(0.107)	(0.110)	(0.114)	
$Top1_{t-1}$>Thr2					-0.218*
					(0.116)
$lgdppk_{t-1}$		-0.014***	-0.021***	-0.021***	-0.018***
		(0.005)	(0.005)	(0.006)	(0.006)
$\left(\frac{i}{y}\right)_{t-1}$			0.193***	0.195***	0.171***
			(0.064)	(0.064)	(0.065)
π_{t-1}				-0.005	-0.003
				(0.024)	(0.024)
_cons	0.054***	0.201***	0.213***	0.217***	0.177***
	(0.011)	(0.052)	(0.051)	(0.055)	(0.056)
Threshold1	-0.8746	-0.8746	-0.8746	-0.8746	-0.8750
Threshold2					-0.8806
F-stat	9.38	11.90	14.41*	14.44 *	174.73***
Num of id<Thr1	66	66	66	66	65
Num of id<Thr2					1
N	445	445	445	445	445
R^2	0.030	0.049	0.068	0.068	0.034
$YearFE$	No	No	No	No	No

Robust Standard errors in parentheses * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$ This table summarizes the results of the panel threshold regression over 1921-2010 for five countries: Australia, Canada, France, Japan and the United States. The dependent variable is the difference of the log of GDP per capita in 1990 international GK$. The exogenous variable is the lagged top 1 percent income share. The control variables are the lagged GDP per capita in log, lagged investment to GDP ratio, and lagged inflation rate. The threshold is the standardized top income share normalized by the average and the standard deviation.

Table A8: Panel threshold regression - 1921-2010 - 5 countries - time fixed effects

	(1) Single1	(2) Single2	(3) Single3	(4) Single4	(5) Single5
$Top1_{t-1}$<Thr1	-0.495*	-0.698***	-0.708***	-0.722***	-0.722***
	(0.254)	(0.261)	(0.261)	(0.264)	(0.264)
$Top1_{t-1}$>Thr1	-0.267	-0.377**	-0.390**	-0.404**	
	(0.170)	(0.172)	(0.174)	(0.176)	
$Top1_{t-1}$>Thr2					-0.404**
					(0.176)
$lgdppk_{t-1}$		-0.040***	-0.041***	-0.044***	-0.044***
		(0.014)	(0.014)	(0.015)	(0.015)
$\left(\frac{i}{y}\right)_{t-1}$			0.053	0.065	0.065
			(0.087)	(0.091)	(0.091)
π_{t-1}				-0.014	-0.014
				(0.031)	(0.031)
_cons	0.111***	0.454***	0.455***	0.479***	0.479***
	(0.037)	(0.122)	(0.122)	(0.133)	(0.133)
Threshold1	-0.8750	-0.8750	-0.8750	-0.8750	-0.8750
Threshold2					-0.8806
F-stat	17.13*	23.97***	24.24 **	24.35 **	172.17***
Num of id<Thr1	66	66	66	66	65
Num of id<Thr2					1
N	445	445	445	445	445
R^2	0.292	0.310	0.310	0.311	0.311
$YearFE$	yes	yes	yes	yes	yes

Robust Standard errors in parentheses * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$ This table summarizes the results of the panel threshold regression over 1921-2010 for five countries: Australia, Canada, France, Japan and the United States. The dependent variable is the difference of the log of GDP per capita in 1990 international GK$. The exogenous variable is the lagged top 1 percent income share. The control variables are the lagged GDP per capita in log, lagged investment to GDP ratio, and lagged inflation rate. The threshold is the standardized top income share normalized by the average and the standard deviation.

Table A9: PSTR: 11 countries 1950-2010

	(1)	(2)	(3)	(4)	(5)
(m, r^*)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)
β_1	-0.72***	-0.593***	-0.592***	-0.575***	-0.564***
	(0.147)	(0.144)	(0.145)	(0.146)	(0.145)
β_2	0.39***	0.326***	0.327***	0.316**	0.253***
	(0.075)	(0.071)	(0.072)	(0.074)	(0.075)
$lgdppk_{t-1}$		-0.019***	-0.025***	-0.020***	-0.015**
		(0.002)	(0.005)	(0.005)	(0.005)
hc_{t-1}			0.012	-0.002	-0.001
			(0.007)	(0.008)	(0.008)
$\left(\frac{i}{y}\right)_{t-1}$				-0.060***	-0.062***
				(0.022)	(0.022)
π_{t-1}					-0.0003***
					(0.00)
γ	3.05	2.69	2.61	2.50	2.46
c	-0.63	-0.43	-0.45	-0.45	-0.49
RSS	0.39	0.34	0.34	0.34	0.33
AIC	-7.428	7.543	-7.54	-7.55	-7.58
N	660	660	660	660	660
R^2	0.041	0.148	0.152	0.161	0.193
ID	11	11	11	11	11
$CountryFE$	Yes	Yes	Yes	Yes	Yes
$YearFE$	No	No	No	No	No

Standard errors in parentheses * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$ This table summarizes the results of the threshold regression for a panel of 11 countries over the period 1950-2010. The dependent variable is the difference of the log of GDP per capita in 1990 international GK $. The exogenous variable is the lagged top 1 percent income share. The control variables are the lagged GDP per capita in log, lagged human capital, lagged investment to GDP ratio, and lagged inflation rate. The threshold is the standardized top income share. The 11 countries are: Australia, Canada, Denmark, Finland, France, Japan, New Zealand, Norway, Sweden, the United Kingdom, and the United States. γ and c are the parameters of the logistic function. LM_F-stat 1 and LM_F-stat 2 are the statistics for the tests for remaining nonlinearity for a number of location parameter $m = 1$ and $m = 2$.

Table A10: PSTR: 11 countries 1950-2010

	(1)	(2)	(3)	(4)	(5)
(m, r^*)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)
β_1	-0.45***	-0.336***	-0.332***	-0.376***	-0.405***
	(0.127)	(0.117)	(0.115)	(0.123)	(0.124)
β_2	0.30***	0.263***	0.258***	0.337***	0.339***
	(0.067)	(0.064)	(0.063)	(0.074)	(0.075)
$lgdppk_{t-1}$		-0.051***	-0.050***	-0.043***	-0.040***
		(0.005)	(0.005)	(0.006)	(0.005)
hc_{t-1}			-0.050	-0.021**	-0.021**
			(0.008)	(0.008)	(0.008)
$\left(\frac{i}{y}\right)_{t-1}$				-0.094***	-0.101***
				(0.021)	(0.022)
π_{t-1}					-0.0001**
					(0.00)
γ	3.25	3.50	3.60	2.97	3.00
c	-0.56	-0.65	-0.64	-0.78	-0.82
RSS	0.24	0.21	0.21	0.199	0.198
AIC	-7.739	-7.87	-7.87	-7.90	-7.90
N	660	660	660	660	660
R^2	0.413	0.489	0.490	0.506	0.509
ID	11	11	11	11	11
$CountryFE$	Yes	Yes	Yes	Yes	Yes
$YearFE$	Yes	Yes	Yes	Yes	Yes

Standard errors in parentheses * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$ This table summarizes the results of the threshold regression for a panel of 11 countries over the period 1950-2010. The dependent variable is the difference of the log of GDP per capita in 1990 international GK $. The exogenous variable is the lagged top 1 percent income share. The control variables are the lagged GDP per capita in log, lagged human capital, lagged investment to GDP ratio, and lagged inflation rate. The threshold is the standardized top income share. The 11 countries are: Australia, Canada, Denmark, Finland, France, Japan, New Zealand, Norway, Sweden, the United Kingdom, and the United States. γ and c are the parameters of the logistic function. LM_F-stat 1 and LM_F-stat 2 are the statistics for the tests for remaining nonlinearity for a number of location parameter $m = 1$ and $m = 2$.