

Productivity and unemployment: a scale-by-scale panel data analysis for the G7 countries

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Abstract

Does productivity growth increase or reduce unemployment? Theoretical and empirical analyses have generally provided mixed results. In this paper we analyze the empirical relationship between productivity and unemployment over different time frames using wavelet analysis. The scale-by-scale results from panel data and nonparametric regressions methods indicate that productivity-unemployment relationship is scale-dependent (change over different time horizons). Specifically, productivity growth creates unemployment in the short and medium terms, but employment in the long run.

Keywords: productivity growth; unemployment; wavelets; panel data.

JEL: C1, C3, C5, E3

1 Introduction

Does productivity growth increase or reduce unemployment? Neither theoretical nor empirical studies provide us with a definitive answer on the issue whether productivity growth is good or bad for employment.

Economic theory so far has been ambiguous about the nexus between productivity and unemployment. Whereas real business cycle models predict a short-run positive effect of technology improvements on employment, sticky-prices new Keynesian models imply opposite short-run effects and a rise in the long-run for employment. Within labor market models, search theories, e.g. in Pissarides (2000) and Aghion and Howitt (1994), predict that the long-run relationship between labor productivity and unemployment is negative when technology is disembodied and positive when it is embodied, respectively.

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Although a number of empirical studies have documented a negative relationship between unemployment and productivity growth at low frequencies (e.g. Bruno and Sachs, 1985, Phelps, 1994, Blanchard *et al.*, 1995, Staiger *et al.*, 2002, and Pissarides and Vallanti, 2007), the issue whether productivity growth is good or bad for employment in the short run is still highly controversial.¹ Moreover, phenomena like the jobless recovery or the recent experience in the US and Europe² have fostered the debate on the existence of a trade-off between unemployment and productivity growth (Gordon, 1997).

Despite the fact that in this literature structural VARs regressions with long-run restrictions represent the most widely used approach, studies arguing the need to investigate co-movements between productivity and unemployment over different time frames have recently emerged (e.g. Tripier 2006, and Chen *et al.*, 2007). Blanchard *et al.* (1995) were the first ones to hint at such a research agenda by stressing that it may be useful to distinguish between the short, medium and long-run effects of productivity growth, as the effects of productivity growth on unemployment may differ at different time scales.³ A similar intuition is also reported in Solow (2000) with respect to the different usefulness of alternative theoretical macroeconomic frameworks in relation to their specific time frames:

At short term scales, I think, something sort of Keynesian is a good approximation, and surely better than anything straight neoclassical. At very long scales, the interesting questions are best studied in a neoclassical framework..... At the five to ten years time scale, we have to piece things together as best as we can, and look for an hybrid model that will do the job Solow (2000, p.156).

The issue of time scales in the context of the relationships between labor market variables has recently been expressed in Landmann (2004, p.35): "the nature of the mechanism that link [unemployment and productivity growth] changes with the time frame adopted" because one needs "to distinguish between an analysis of the forces shaping long-term equilibrium paths of output, employment and productivity on the one hand and the forces causing temporary deviations from these equilibrium paths on the other hand".

¹See the papers by Galí, (1999), Galí and Rabanal, (2005), Francis and Ramey, (2005), and Basu *et al.*, (2006) which provide evidence of contrasting short-run and long-run effects of productivity growth for (un)employment.

²Whereas the increase in productivity growth in the US in the second half of the 90's has been associated with low and falling unemployment (Staiger *et al.* 2002), in Europe productivity growth appears to have increased unemployment.

³Most of the attention of economic researchers working on productivity has been devoted to measurement issues and to resolve the problem of data consistency, as there are many different approaches to the measurement of productivity linked to the choice of data, notably the combination of employment, hours worked and GDP (see for example the OECD Productivity Manual, 2001).

In this paper we propose a "scale-based" panel data approach that allows to analyze separately the short-, medium- and long-term effects of the changes of productivity for unemployment within a panel data framework.⁴ The proposed approach is based upon the multiresolution decomposition properties of the wavelet transform that provides a time-scale representation of a given signal by describing its time evolution on a scale-by-scale basis. In particular, wavelet analysis attains an optimal trade-off between time and frequency resolution levels (Lau and Weng, 1995, Mallat, 1989) because of its ability to decompose a given time series into different components, each with a resolution matched to its scale.

Such a multiscale decomposition approach provides a natural framework for the analysis of relationships that, like the one between productivity growth and unemployment, are likely to exhibit frequency-dependent behavior. Specifically, after decomposing the two variables into their time-scale components through the maximum overlap discrete wavelet transform (MODWT), we gather data for each time scale component into separate panel data sets and then estimate the relationship between productivity growth and unemployment on a "scale-by-scale" basis using panel data regression analysis. The results provide evidence of contrasting short-run, medium-run and long-run effects of productivity growth on unemployment. In particular, at scales corresponding to the medium-run and the business cycle frequency range there is evidence of a positive relationship between productivity and unemployment, whereas in the long-run we can observe the opposite.

In addition, using nonparametric regression methods we check whether panel data regressions inappropriately restrict coefficients to be the same across countries. Nonparametric analysis provides robust evidence about the negative relationships between the long-run components of labor productivity and the unemployment rate across the G7 countries. Moreover, the evidence at scales corresponding to the higher frequencies provides support to the hypothesis of a trade-off between unemployment and productivity growth. Finally, a robustness check is performed by applying both parametric and nonparametric regression analysis to US quarterly data between 1948 and 2013 to confirm the accuracy and reliability of the results obtained with the "scale-based" panel data approach.

The paper proceeds as follows. In section 2 we describe the methodology and the dataset used in the paper. Then, in section 3, after decomposing productivity growth and unemployment through the MODWT we examine their scale-by-scale relationships using panel data regression analysis for the G7 countries over the period 1962-2012. Section 4 shows the results

⁴Recently developed panel unit root and panel cointegration techniques also allows to estimate the long-run and short-run relationships between variables (see Im *et al.*, 2003, and Pedroni, 2001, respectively).

from nonparametric regression analysis using the *loess* method. In section 5 in order to check the robustness of our findings we replicate the analysis performed in the previous sections using US quarterly data from 1948 to 2013. Finally, section 6 concludes the paper.

2 Methodology and dataset

The economic intuition supporting the application of time-frequency domain techniques is that economic and financial processes can be the result of decisions of agents with different, sometimes very different, time horizons.⁵ The labor market provides an example of a market in which the agents involved, firms and workers (through unions), interact at different time horizons. Hence, both the time horizon of economic decisions and the strength and direction of economic relationships among labor market variables, *i.e.* wages, prices and unemployment, can vary across time scales.⁶ As a result, the long run effects of technological decisions may be different from medium run effects, and both may be different from short run reactions. In the medium run, new technology is likely to be labor reducing, and thus adding to unemployment,⁷ as was visible in Europe during the 1990s. In the long run, however, new technology replacing labor increases productivity, thereby making firms and the economy more competitive which in turn will reduce unemployment.⁸ Also to consider are the effects of those product innovation processes which employ workers previously unemployed or employed by firms competing with process innovating firms.

2.1 Methodology

In such a context methods allowing to separate aggregate data into different frequency components can be very appealing. Although spectral methods have been by far the most important filter processing tool in economics for many years (e.g. Engle, 1974, 1978, and Lucas, 1980), the role of wavelets in economic and financial empirical literature has been rapidly expanding,⁹

⁵For example, in financial markets the presence of heterogeneous agents with different trading horizons may generate very complex patterns in the time-series of economic and financial variables (e.g., Muller *et al.*, 1995, and Lynch and Zumbach, 2003).

⁶E.g. in Gallegati *et al.* (2009, 2011) where wavelet analysis is applied to the wage Phillips curve for the US.

⁷A statement like this goes back to David Ricardo who has pointed out that if machinery is substituted for labor unemployment is likely to increase.

⁸This point is made clear in a simple text book illustration by Blanchard (2005).

⁹After the first applications of wavelet analysis in economics and finance provided by Ramsey and his coauthors (e.g. Ramsey and Zhang, 1995, 1996, Ramsey *et al.*, 1995, Ramsey and Lampart, 1998a, 1998b), the number of wavelet applications in economics has been rapidly growing in the last few years as a result of the increasing interest in this new tool to study economic relationships at different time scales (see Gençay *et al.*, 2001, 2003, 2005, 2010, Kim and In, 2005, Fernandez, 2005, In and Kim, 2006, Crowley and

mainly because of the serious drawbacks affecting the Fourier transform. First, it has only frequency resolution but not time resolution and, since the transformation to the frequency domain does not preserve the time information, it is impossible to determine when a particular event took place. Second, since spectral decomposition methods perform a global analysis, in Fourier analysis a single disturbance can affect all frequencies for the entire length of the series as all projections are global, and thus the signal need to be assumed homogeneous over time. Such a feature restricts the usefulness of the Fourier transform to the analysis of stationary processes, whereas most economic and financial time series display frequency behavior that changes over time, *i.e.* they are nonstationary (Ramsey and Zhang, 1995).

Wavelets provide a unique tool for the analysis of economic relationships over different time frames and may overcome the main problems evidenced by Fourier analysis.¹⁰ The application of the discrete wavelet time transform allows the researcher to decompose each variable into a set of different components, each associated to a particular frequency range, where the variation in each variable has been restricted to the indicated specific scale (e.g. Ramsey and Lampart, 1998a, 1998b, Kim and In, 2005, and Gallegati *et al.*, 2009, 2011). The wavelet transform uses a basis function that is similar to a sine and cosine function in that it also oscillates around zero, but differ because it is well-localized both in the time and the frequency domain, as wavelets are constructed over finite intervals of time. Therefore, with respect to other filtering methods wavelets are well-suited for the analysis of non-stationary signals since the wavelet transform performs a local, and not global, decomposition. Indeed, much of the usefulness of wavelet analysis has to do with its flexibility in handling a variety of nonstationary signals.¹¹ Wavelets, their generation, and their potential use are discussed in intuitive terms in Ramsey (2010), while Gençay *et al.* (2002) provide many interesting example for economics and finance.¹²

The detailed analysis presented above can be simply summarized. The theoretical model to be considered must allow for the interaction between planning horizon and calendar time. The planning horizon is equivalent to the choice of scale for the reaction between productivity and unemployment. Clearly, this relationship will not be stationary and may well change over time. Consequently, the appropriate tool of analysis is not Fourier series, but

Mayes, 2008, Gallegati, 2008, Ramsey *et al.*, 2010, In *et al.*, 2011 and Xu *et al.*, 2014).

¹⁰For details about mathematical similarities and differences between Fourier and wavelet transforms see Strang (1993).

¹¹Wavelets, in opposition to time domain and frequency domain analyses, consider nonstationarity an intrinsic property of the data rather than a problem to be solved by pre-processing the data. In particular, since the base scale includes any non-stationary components, the data need neither be detrended nor differenced (Schleicher, 2002).

¹²Percival and Walden (2000) provide a more technical exposition with many examples of the use of wavelets in a variety of fields, but not in economics.

wavelets. The reason is clear. Fourier analysis provides efficient estimates of constituent frequencies only in the case where frequencies are stationary and the composition of the frequencies is fixed. This is not the case in this article. Wavelets provide an orthogonal decomposition over scale and time or space which is what is needed in this situation. In short, we are allowing for variation in the results depending on planning horizon (scale) and calendar time.

2.2 Dataset

While econometric works, especially those in the RBC/DSGE tradition, have studied the effects of productivity growth on employment using a VAR methodology, we want to focus on the nexus between productivity growth and unemployment given the importance of the unemployment rate in the preference system of policy makers. Although unemployment rates may be impacted by long run forces such as population growth, demographic shifts, and changing labor market participation rates of certain segments of the population, one might presume that the demand side of labor, the offered employment by firms, is the most essential factor for driving the unemployment rate. The visual inspection of the US long-run components of unemployment and employment presented in Figure 1 confirms that the two variables have opposite trending behavior throughout the sample, with employment slightly leading the unemployment rate.¹³

Figure 1 about here

We use annual data of labor productivity and unemployment for the G7 countries from 1962 to 2012. Labor productivity is defined as GDP per hour worked in 1990 US converted at Geary Khamis PPPs (source: The Conference Board Total Economy Database, January 2013, <http://www.conference-board.org/data/economydatabase>). Unemployment rates data refer to the civilian working-age population and are from the US Bureau of Labor Statistics (BLS) International Labor Comparisons program (source: Division of International Labor Comparisons, U.S. Bureau of Labor Statistics, www.bls.gov/ilc). Foreign-country data are mainly based on labor force surveys from national statistical agencies¹⁴ and are adjusted to a common framework by using the concepts adopted by the U.S. Current Population Survey (CPS).

Because of the practical limitations of DWT wavelet analysis is generally performed by applying the *maximal overlap discrete wavelet transform*

¹³The main difference between the low frequency components of the two series is limited to the hump-shaped path of unemployment rate, which is generally explained by the baby boomer demographic effect (Francis and Ramey, 2009). We also find very similar evidence for the relationship between the long-term components of hours worked and unemployment.

¹⁴They also arise from the Organization for Economic Cooperation and Development (OECD) and the Statistical Office of the European Communities (EUROSTAT)

(MODWT), a non-orthogonal variant of the classical discrete wavelet transform that, unlike the DWT, is i) translation invariant, as shifts in the signal do not change the pattern of coefficients, ii) can be applied to data sets of length not divisible by 2^J and iii) returns at each scale a number of coefficients equal to the length of the original series. The wavelet filter used in the decomposition is the Daubechies least asymmetric (LA) wavelet filter of length $L = 8$, that is $LA(8)$, based on eight non-zero coefficients (Daubechies, 1992), with reflecting boundary conditions.

The application of the MOWDT to the annual percent change of labor productivity and the percent level of unemployment rate with a level of decomposition $J = 3$ produces one vector of scaling coefficients s_3 , describing the underlying smooth behavior of the data at the coarse scale, and three vectors of details coefficients d_3 , d_2 and d_1 , representing progressively finer scale deviations from the smooth behavior. Then, we can reconstruct the detail and smooth components of the original signals through the synthesis or reconstruction operation that reassembles the original signal from the wavelet and scaling coefficients by using the inverse stationary wavelet transform.¹⁵ For a 3-level decomposition we get, for each variable, three wavelet details vectors D_1 , D_2 , D_3 that, since we use annual data, captures oscillations with periods of 2-4 years (D_1), 4-8 years (D_2), 8-16 years (D_3), and one wavelet smooth vector, S_3 which captures oscillations with a period longer than 16 years corresponding to the low-frequency components of a signal.¹⁶

3 Panel data estimation on a scale-by-scale basis

The detail and smooth components obtained by applying MODWT to each country variable have been stacked into separate panel data sets, one for each time scale component. We get 4 panel data sets composed by 7 cross-section units (G7) with 51 observation each (1962-2012), where any single panel includes components corresponding to a specific frequency band. For estimating the productivity-unemployment relationship on a "scale-by-scale" basis we estimate for each panel a fixed effects model,¹⁷ that is a model with individual-specific effects:

$$ur[S_J]_{it} = \alpha_{J,i} + \beta_J lp[S_J]_{it} + \epsilon_{J,it} \quad (1)$$

¹⁵Since the J components obtained by the application of MODWT are not orthogonal, they do not sum up to the original variable.

¹⁶With annual data detail levels D_1 and D_2 , roughly correspond to the standard business cycle time period (Stock and Watson, 1999), while the medium-run component is associated to level D_3 .

¹⁷With cross-sectional units such as G7 countries the individual effects can be treated as fixed constant parameters rather than to be drawn from a distribution as in the random effect model.

and

$$ur[D_j]_{it} = \alpha_{j,i} + \beta_j lp[D_j]_{it} + \epsilon_{j,it} \quad (2)$$

where $ur[S_J]_{it}$, and $lp[S_J]_{it}$ represent the smooth components of the unemployment rate and labor productivity growth for the country i at time t , $ur[D_j]_{it}$, and $lp[D_j]_{it}$ represent the detail components of the two variables at each j scale, $j=1,2,\dots,J$, for the country i at time t , and α_i individual effects with $i = 1, \dots, N$.

Table 1: "Scale-by-scale" panel regression analysis (1962-2012)

$$ur_{j,it} = \alpha_{j,i} + \beta_j \Delta lp_{j,it} + \epsilon_{j,it}$$

	Aggregate	S_3	D_3	D_2	D_1
β_j	-0.599 (-4.22)	-0.968 (-4.16)	<i>0.360</i> (1.70)	0.191 (2.48)	0.002 (0.28)
Adj. R^2	0.4170	0.6188	0.0399	0.0809	0.0001

Note: Fixed individual effects estimation of the productivity-unemployment relationship for the G7 countries with aggregate and scale-based panel datasets. t-statistics in parenthesis: 5% significance level in bold, 10% significance level in italic.

In Table 1 we present the results of panel regression estimates at different time scales using the fixed effect estimator. For the sake of comparison, the first column of Table 1 reports also the results with aggregate data.¹⁸

Panel data estimate of the productivity-unemployment relationship using aggregate data provides evidence of a significant negative relationship. Nonetheless, when the same relationship is examined at different frequency bands the comparison among regressions at different scale levels indicates that the effects of productivity growth on the unemployment rate differ widely in terms of estimated sign and size effects. First, there is clear evidence of sign reversal: the positive relationship displayed at scale levels D_2 and D_3 , corresponding to the longest business cycle frequencies (4 to 8 years) and the medium run, respectively, becomes negative in the long-run.¹⁹ Second, as regards the estimated size effect of productivity growth

¹⁸Since the standard time domain analysis implicitly assigns all frequencies equal weight, the estimated relationships between aggregate time series can be considered estimates "averaged" over all time scales (Ramsey, 1999). As a consequence, the "true" economic relationship between variables can be detected more easily at the disaggregated (scale) level rather than at the usual aggregate level since the effect of each regressor can be masked by averaging.

¹⁹The finding of a negative relationship between the long-term components of productivity growth and unemployment is not new. A negative comovement at low frequencies is documented in Staiger *et al.* (2002) and Ball and Moffit (2002), where the trending

on unemployment, we find that it tends to increase as the scale level increases, with the maximum value in the long-run where a 1% rise in the long-run productivity growth rate decreases the unemployment rate by a corresponding percent value.²⁰

The results provided in this section are fully in line with those obtained in previous papers estimating this relationships over different time frames. For example, Tripier (2006) studying co-movements of productivity and hours worked at different frequency components through spectral analysis finds that such co-movements are negative in the short and long run, but positive at business cycle frequencies. In addition, Chen *et al.* (2007), by disaggregating data into their short and long-term components and using two different econometric methods, i.e. Maximum Likelihood (ML) and structural VAR, find that productivity growth affects unemployment positively in the short run and negatively in the long run.

4 Testing for coefficient stability across countries using *loess*

In this section we try to address two questions: the presence of non-linearity and whether panel data regressions unduly restrict coefficients to be the same across countries, so that it might be better to model countries separately.²¹ Specifically, we apply nonparametric regression analysis, a methodology that allows us to explore the robustness of the relationship between labor productivity growth and unemployment without making any *a priori* explicit or implicit assumption about the form of the relationship.²²

There are several approaches available to estimate nonparametric regression models,²³ with most of these methods assuming that the nonlinear function of the independent variable to be estimated by the procedure is a

behavior of productivity growth is called for in the explanation of low and falling inflation combined with low unemployment experienced by the US during the second half of the 1990s, and also in Muscatelli and Tirelli (2001) for several G7 countries. Recently, a negative long-run connection between productivity growth and unemployment has also been obtained in Schreiber (2009) using a co-breaking approach and in Miyamoto and Takahashi (2011) using the band-pass filtering approach.

²⁰This estimated magnitude of the impact of growth on unemployment is in line with those obtained in previous studies. For example, Pissarides and Vallanti (2007) using a panel of OECD countries estimate that a 1% decline in the growth rate leads to a 1.3–1.5% increase in unemployment.

²¹In Muscatelli and Tirelli (2001) the relationship between productivity growth and unemployment is negative for several G7 countries and not significant for others.

²²The traditional nonlinear regression model introduces nonlinear functions of dependent variables using a limited range of transformed variables to the model (quadratic terms, cubic terms or piecewise constant function). An example of a methodology testing for nonlinearity without imposing any *a priori* assumption about the shape of the relationship is the smooth transition regression used in Eliasson (2001).

²³See Fox (2000a, 2000b) for a discussion on nonparametric regression methods.

smooth continuous function. One such model is the locally weighted polynomial regression, i.e. *loess*, pioneered by Cleveland (1979). This procedure fits the model $y = f(x_1, \dots, x_k) + \epsilon$ nonparametrically, that is without assuming a parametric form for $f(x_1, \dots, x_k)$. The low-degree polynomial, generally first or second degree, and thus either locally linear or locally quadratic, is fit using weighted least squares, with data points weighted by a smooth function whose weights decrease as the distance from the center of the window increases. The value of the regression function is obtained by evaluating the local polynomial at each particular value of the independent variable, x_i , where a fixed proportion of the data, called the *span* of the local regression smoother (or the smoothing parameter), is included in each given local neighborhood and the fitted values are then connected in a nonparametric regression curve. The main advantage of the local regression (*loess*) method is that it does not require the specification of a function to fit a model to all of the data in the sample. In addition, it provides robust fitting when there are outliers in the data, supports multiple dependent variables and computes confidence limits for predictions when the error distribution is symmetric, but not necessarily normal.²⁴

Figures 2 to 5 about here

In Figures 2 to 5 we report the scatter plots of unemployment (y-axis) against productivity growth (x-axis) for each country at different scale levels, from S_3 to D_1 . In each panel a smooth fitted line is drawn using the *loess* method (see Cleveland, 1993) with the smoothed values obtained using a first-degree polynomial and a smoothing parameter value of 0.5.²⁵ These lines can be used to reveal the nature of the estimated relationship between the dependent (unemployment rate) and the response variable (labor productivity growth).

The smooth lines superimposed in the scatterplots in Figures 2 to 5 indicate that countries' relationship between productivity and unemployment is not uniform neither within nor across scales. Indeed, relevant differences emerge at shorter scales, *i.e.* D_2 and D_3 , especially between the "Anglo-Saxon" countries, *i.e.* Canada, the UK and the US, and the G7 European countries plus Japan. Whereas the first group of countries mostly display a positive relationship at both scales, the latter shows a positive relationship only at some scales (France at scale D_2) or for limited periods of time (Italy at scales D_2 and D_3). On the other hand, there is a quite strong uniform behavior across countries with respect to the long-run relationship which is mostly negative.

²⁴On the other hand, *loess*, being a method that fits models to localized subsets of the data, requires reasonably large, densely sampled datasets in order to produce good models.

²⁵We use different smoothing parameters, but our main findings do not show excess sensitivity to the choice of the span in the loess function within what appear to be reasonable ranges of smoothness (*i.e.* between 0.4 and 0.8).

In sum, the analysis of the nonparametric fitted functions provides evidence that the negative relationships between the long-run components of labor productivity and the unemployment rate is fairly robust across the G7 countries. On the other hand, the evidence at scales corresponding to frequencies larger than 4 years is much more sparse and tends to support the hypothesis of different trade-offs between unemployment and productivity growth for different groups of countries. In particular, such different reactions of unemployment to productivity growth are consistent with existing differences in labor market institutions and regulations between the group of Anglo-Saxon countries that have greater labour market flexibility compared to countries having more rigid labour markets like the European G7 countries.

5 Robustness check

Labor market data need to be adjusted to a common conceptual framework in order to be used in international comparisons. This condition, particularly relevant for the validity of international comparisons of labor productivity, can severely limit the number of observations in a sample. Since wavelet analysis can be considered a very demanding method in terms of data, in the sense of requiring a large amount of regularly sampled data, and we are well aware that the reliability of the results obtained in the previous section can be seriously questioned on the basis of the small number of observations of our sample, in this section we examine the robustness of the relationship between productivity and unemployment using observations measured at a different time interval and spanning a different sample period. In particular, we use quarterly data on labor productivity and unemployment for the U.S. from 1948:2 to 2013:3.²⁶ The comparison with the results reported for the U.S. in the previous section can provide a reliable check of the robustness and sensitivity issues associated to our findings. The robustness check is performed with respect to different wavelet filters and testing the stability of the relationship over time.

5.1 Frequency interpretation of time scale decomposition

Using quarterly in spite of annual data has no notable implication for the wavelet transform outcomes except for the different frequency interpretation of the crystals obtained from the multiresolution decomposition analysis. Table 1 reports how the frequency interpretation of the multiresolution decomposition scale levels changes using quarterly and annual data for detail

²⁶Labor productivity is defined as output per hour of all persons in the Nonfarm Business Sector, Index 2005=100 (series ID:OPHNFB). Index values are transformed into their growth rates as $400 * \ln(x_t/x_{t-1})$. Unemployment rate is defined as percent Civilian Unemployment Rate. Data are from the Bureau of Labor Statistics.

level components D_1 , D_2 , D_3 , D_4 and D_5 . In particular, using quarterly data the first detail level D_1 captures oscillations between 1/2 and 1 years, while details D_2 , D_3 , D_4 and D_5 capture oscillations with a period of 1-2, 2-4, 4-8 and 8-16 years, respectively.²⁷ Finally, the smooth component S_5 captures oscillations with a period longer than 16 years and corresponds to the S_3 component obtained using annual data.

Table 2: Frequency interpretation of MRD scale levels in years

Scale level J	Detail level D_j	Quarterly data	Annual data
1	D_1	1/2-1	2-4
2	D_2	1-2	4-8
3	D_3	2-4	8-16
4	D_4	4-8	16-32
5	D_5	8-16	32-64

Figure 6 about here

The smooth and detail components obtained from the reconstruction process take the form of non-periodic oscillating waves representing the long-term trend and the deviations from it at an increasing level of detail. According to Ramsey (2002) the visual inspection of these oscillatory components between pairs of variables provides an excellent exploratory tool for discovering time varying delays or phase variations between variables.²⁸

In Figure 6 we plot the smooth component S_5 and the highest level deviations from the smooth component, *i.e.* D_5 , D_4 and D_3 , of the unemployment rate (black lines) and labor productivity growth (grey lines) as a sequence of pairs of time series. The visual inspection of the long-run components indicate a clear anti-phase relationship between the two variables, with productivity growth slightly leading the unemployment rate.²⁹ The pattern displayed in the top right panel of Figure 6 shows that the components at the D_5 scale level are mostly in phase, with unemployment slightly

²⁷With quarterly data detail levels D_1 and D_2 , represent the very short-run dynamics of a signal (and contains most of the noise of the signal), levels D_3 and D_4 roughly correspond to the standard business cycle time period (Stock and Watson, 1999), while the medium-run component is associated to level D_5 .

²⁸A standard assumption in economics is that the delay between variables is fixed, but, as evidenced in Ramsey and Lampart (1998a and 1998b) and in Gallegati and Ramsey (2013), the phase relationship may well be scale dependent and vary continuously over time. By examining the phase relationship in a bivariate context we can obtain useful insights on the timing (lagging, synchronous or leading) of the linkage between variables as well as on the existence of a fixed or changing relationship.

²⁹This leading behavior is consistent with the hypothesis that changes in productivity growth are likely to affect the unemployment rate through wage aspiration adjustments (Stiglitz, 1997).

leading productivity growth. Nonetheless, the plot also shows that the two series have been moving into anti-phase at the beginning of the nineties, as a consequence of a shift in their phase relationship, and then have been moving in-phase again at the end of the sample. At the D_4 scale level productivity and unemployment are in-phase throughout the sample with the exception of the sixties. Finally, at the lowest scale levels, from D_3 to D_1 , the most notable feature is represented by the different amplitude between productivity growth and unemployment components, with the first displaying a much larger amplitude than the latter. This pattern suggests that a well known, and very interesting, feature of aggregate productivity growth quarterly data, that is its high volatility, can be ascribed to the specific pattern of the high frequency components.³⁰

5.2 Testing robustness across methods and over time

As in the previous section, after decomposing the regression variables into their time scale components using the MOWDT, we estimate a sequence of least squares regressions using

$$ur[S_J]_t = \alpha_J + \beta_J lp[S_J]_t + \epsilon_t \quad (3)$$

and

$$ur[D_j]_t = \alpha_j + \beta_j lp[D_j]_t + \epsilon_t \quad (4)$$

where $ur[S_J]_t$, and $lp[S_J]_t$ represent the components of the variables at the longest scale, and $ur[D_j]_t$, and $lp[D_j]_t$ represent the components of the variables at each j scale, with $j=1,2,\dots,J$.

Table 3 shows the least squares results at different scale levels using US quarterly data from 1948:2 to 2013:3. Column (i) present the results obtained using the Daubechies LA(8) wavelet filter with reflecting boundary condition, columns (ii) reports estimation results using the same wavelet filter, LA(8) with a different boundary condition, that is circular, and, finally, column (iii) shows the results using the Haar wavelet filter. In this way we can test the sensitivity of our results to different boundary conditions and different wavelet filters.³¹ The results in Table 3 indicate that although at the aggregate level the relationship between unemployment rate and productivity is not significant,³² when the same relationship is examined at multiple scales it turns out to be statistically significant at (almost) any scale level. Moreover, the scale-by-scale regressions indicate that the effect of productivity on the unemployment rate may also differ across scales in

³⁰It is just to overcome these problems due to the volatility that studies generally tend to measure underlying productivity trends by calculating annual average rates of growth.

³¹We thank an anonymous referee for drawing our attention to this point.

³²The estimated coefficient of labor productivity growth for the aggregate relationships is .0261 with a t-statistic .82.

Table 3: *Time scale regression analysis for the US (1948:2-2013:3)*

$$ur[D_j]_t = \alpha_j + \beta_j lp[D_j]_t + \epsilon_t$$

Scale	Wavelet filter	LA(8) reflective	LA(8) circular	Haar
		(i)	(ii)	(iii)
S_5	β_6	-1.8606	-1.6874	-1.6198
	t-stat	(-16.08)	(-14.71)	(-15.08)
	\overline{R}^2	0.8171	0.7609	0.7731
	S.E.	0.5240	0.5139	0.4258
D_5	β_5	0.6582	0.0357	-0.0756
	t-stat	(4.42)	(0.17)	(-0.35)
	\overline{R}^2	0.2166	0.0006	0.0024
	S.E.	0.4663	0.6584	0.5008
D_4	β_4	0.5455	0.4878	0.4463
	t-stat	(9.86)	(9.61)	(6.05)
	\overline{R}^2	0.4711	0.4710	0.3282
	S.E.	0.4040	0.3832	0.3372
D_3	β_3	0.1943	0.2044	0.2014
	t-stat	(8.94)	(8.03)	(6.59)
	\overline{R}^2	0.3950	0.3522	0.2966
	S.E.	0.2639	0.3111	0.2603
D_2	β_2	0.0278	0.0339	0.0428
	t-stat	(2.75)	(3.40)	(3.77)
	\overline{R}^2	0.0428	0.0586	0.0653
	S.E.	0.1586	0.1710	0.1741
D_1	β_1	-0.0094	-0.0090	-0.0070
	t-stat	(-4.43)	(-4.32)	(-3.11)
	\overline{R}^2	0.0787	0.0321	0.0132
	S.E.	0.0692	0.1055	0.1125

Note: S.E. is the regression standard error.

terms of sign and estimated size effect. At the shortest scales, D_1 and D_2 , although the relationship is statistically significant, the estimated size effect of productivity growth on unemployment is negligible. At scales corresponding to business cycle frequencies, *i.e.* D_3 and D_4 , the relationship is positive and highly significant, with an estimated size effect larger at D_4 than at D_3 and the statistical significance that lowers at the scale corresponding to the medium run, D_5 .³³ Finally, at the smooth scale level, S_5 , the size and significance of the estimated coefficient indicates a strong negative relationship between productivity growth and unemployment in the long-run. A 1% increase in the long-run productivity growth rate lowers the unemployment rate by a value between 1.62% and 1.86%. As evidenced by the values reported in columns (i) to (iii) these results are robust to different boundary conditions, circular vs reflective, as well as to different wavelet filters, LA(8) vs Haar.

Figures 7 and 8 about here

Finally, we check for the stability of the "scale-by-scale" results over time by applying nonparametric regression analysis to different sub-samples, *i.e.* 1948:2-1969:4, 1970:1-1991:4 and 1992:1-2013:3.³⁴ The scatter plots in Figures 7 and 8 report the unemployment-productivity relationship at different scale levels (sort by row) for different sub-periods (sort by column) using the *loess* method.³⁵ Specifically, in Figure 7 the top left panel shows the scatter plot at level S_5 for the 1948:2-1969:4 period, and the bottom right panel the scatter plot at the D_4 level for the 1992:1-2013:3 period. Similarly, in Figure 8 the top left panel shows the scatter plot at level D_3 for the 1948:2-1969:4 period, and the bottom right panel the scatter plot at the D_1 level for the 1992:1-2013:3 period. The analysis of the nonparametric fitted functions in Figures 7 and 8 largely confirm our main findings, that is a positive relationship between labor productivity and unemployment at scales corresponding to business cycles and medium-term frequencies and a negative relationship between productivity and unemployment at the longest scale level. The only relevant exceptions refer to the relationship at the S_5 level in the 1948:2-1969:4 period and at the D_5 level in the 1992:1-2013:3 period.

In sum, the main findings stemming from the panel data approach are confirmed by the robustness check performed in this section: the effects of productivity growth on unemployment are frequency-dependent. In the long

³³At the scale level D_5 the difference in estimation results are related to different estimated values at the boundaries. Indeed, after excluding the observations at the boundaries the estimated coefficient values in columns (ii) and (iii) are very close to those reported in column (i), and so it is for the significance tests and statistics.

³⁴The partition divides the whole sample into three sub-samples, each having roughly equal size.

³⁵The smooth plots represented by the solid lines depict the *loess* fit using a smoothing parameter value of .5 (the results are robust to different smoothing parameters).

run an increase in productivity releases forces that stimulate innovation and growth in the economy and thus determine a reduction of unemployment. Nonetheless, these positive effects are partially offset by the negative effects at intermediate and business cycle time scales where productivity gains determine an increase in unemployment.

6 Conclusion

In this paper, following several studies arguing that the nature of the relationship linking the two variables can change with the time frame adopted (Landmann, 2004, Tripier, 2006, and Chen *et al.* 2007), we study the relationship between productivity growth and unemployment for the G7 countries using wavelet analysis. Wavelets allow for a detailed exploration of possible time-varying dynamics linking these two variables over different time frames in a unified framework. The analysis is performed by applying panel estimation and *loess* regression methods on a "scale-by-scale" basis for G7 countries over the period 1970-2010.

The main finding of the paper is that unemployment is positively associated with productivity growth in the short and medium run, but negatively in the long term, a results that is consistent with what already found in the literature using a number of different econometric methodologies. A robustness check performed using quarterly data for the US between 1952 and 2010 confirms the reliability of the frequency-dependent pattern of the productivity-unemployment relationship obtained in the international comparison.

How can we interpret the main findings of this paper in terms, for example, of the RBC vs NK debate? As to the controversial prediction of the RBC models that employment is rising with positive productivity shocks, the critics (such as Basu *et al.*, 2006) are presumably correct to state a non-significant relationship between technology shocks and employment, or even a negative relationship of those variables. So the postulate of flexible-price RBC models of a positive relationship between productivity and employment seems to be incorrect in the short and medium run, but, given our results, is likely to hold on a long time scale. Otherwise, short-run findings are consistent with the predictions generated by NK sticky-price models. The result that a Keynesian model explains short-run behavior, but a neoclassical model the long-run was intuited by Solow (2000, p.156), and represent the theoretical underpinning in Gali (1999), Basu *et al.* (2006), and many other empirical papers in this literature.

As regards search and matching theories of the labor market (e.g. Aghion and Howitt, 1994, Mortensen and Pissarides, 1998, Pissarides, 2000)³⁶ the

³⁶Theoretical predictions of the overall impact of productivity growth on unemployment depend on the relative strength of the "capitalization" and "creative destruction" effects.

empirical evidence provided by wavelet analysis can be interpreted as supportive of the assumption that the "creative destruction" effect dominates over the "capitalization" effect at short- to medium-term scales, whereas the "capitalization" effect dominates at the longest scales. This result is consistent with the different time horizons of "capitalization" and "creative destruction", and their associated effects on job creation and jobs destruction, respectively. Indeed, a firms' time horizon when creating jobs can be very long, and definitely much longer than a firm's horizon when destroying job classifications, because job creation involves computing the expected present discounted value of future profits from new tasks. To summarize, what emerges is a complex picture of the relationship in which the aggregate effect is simply the interaction of the relative strength of effects having different strengths at different time horizons.

The main implication for policy actions is that policies aiming at increasing long-term productivity should not be contrasted on the ground that the structural adjustment process following technological improvements can be costly in terms of jobs creation in the short-run. When Thomas More (Utopia, 1516) was asserting: sheep are eating men, he was, in the short run, right. Due to agricultural innovations, profits in the primary sector were rising, less labor force was employed in agriculture and more lands were devoted to pastureland. People had to "invent" new jobs, *i.e.* people were stimulated into creating new products that the new technology made possible.

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In particular, we should observe a positive relationship between productivity growth and unemployment if the creative destruction effect dominates over the capitalization effect, and vice versa.

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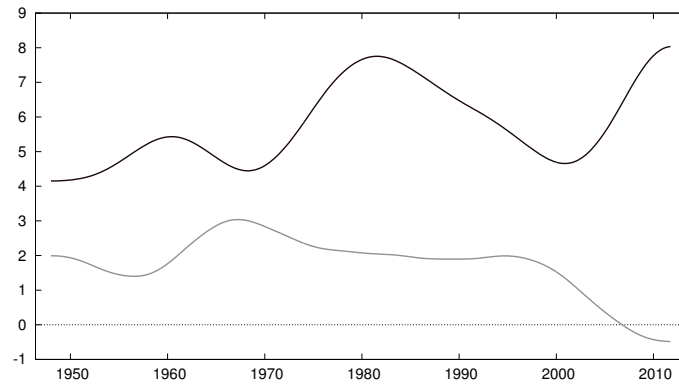


Figure 1: Trend components of unemployment (black thick lines) and employment (grey thin lines)

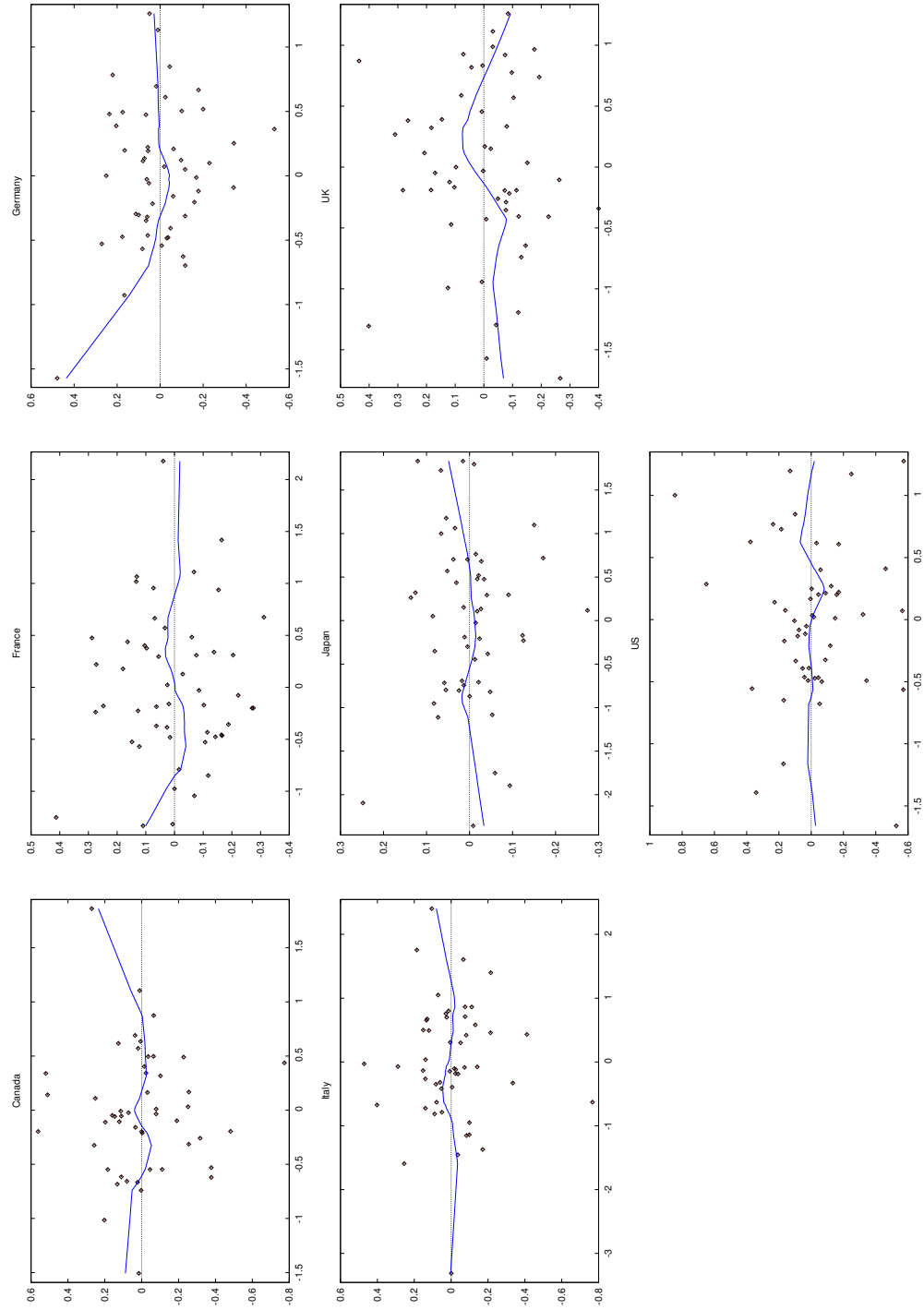


Figure 2: Scatter plot and loess fit (solid line) of labor productivity and unemployment rate for the D_1 components (2-4 years)

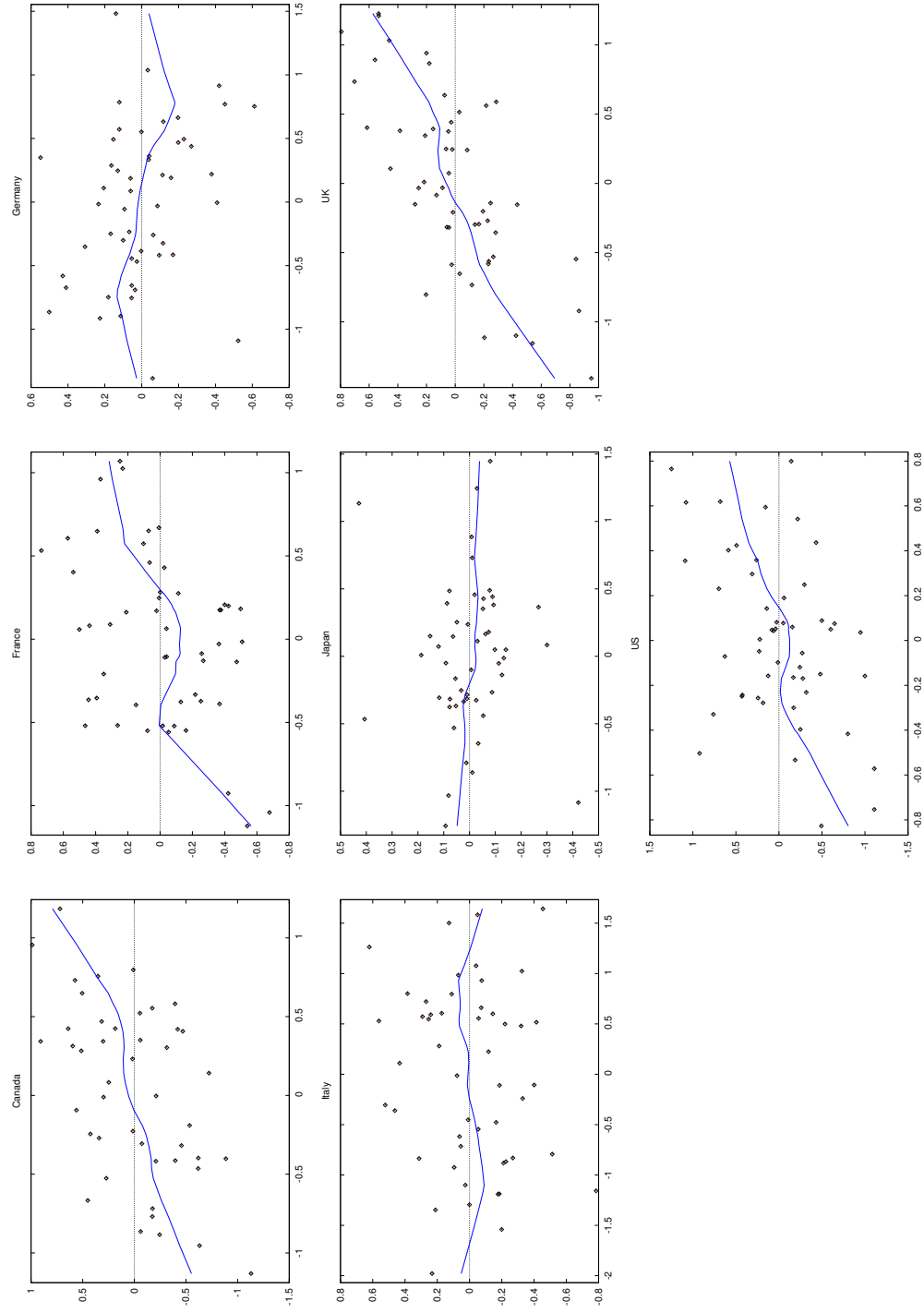


Figure 3: Scatter plot and loess fit (solid line) of labor productivity and unemployment rate for the D_2 components (4-8 years)

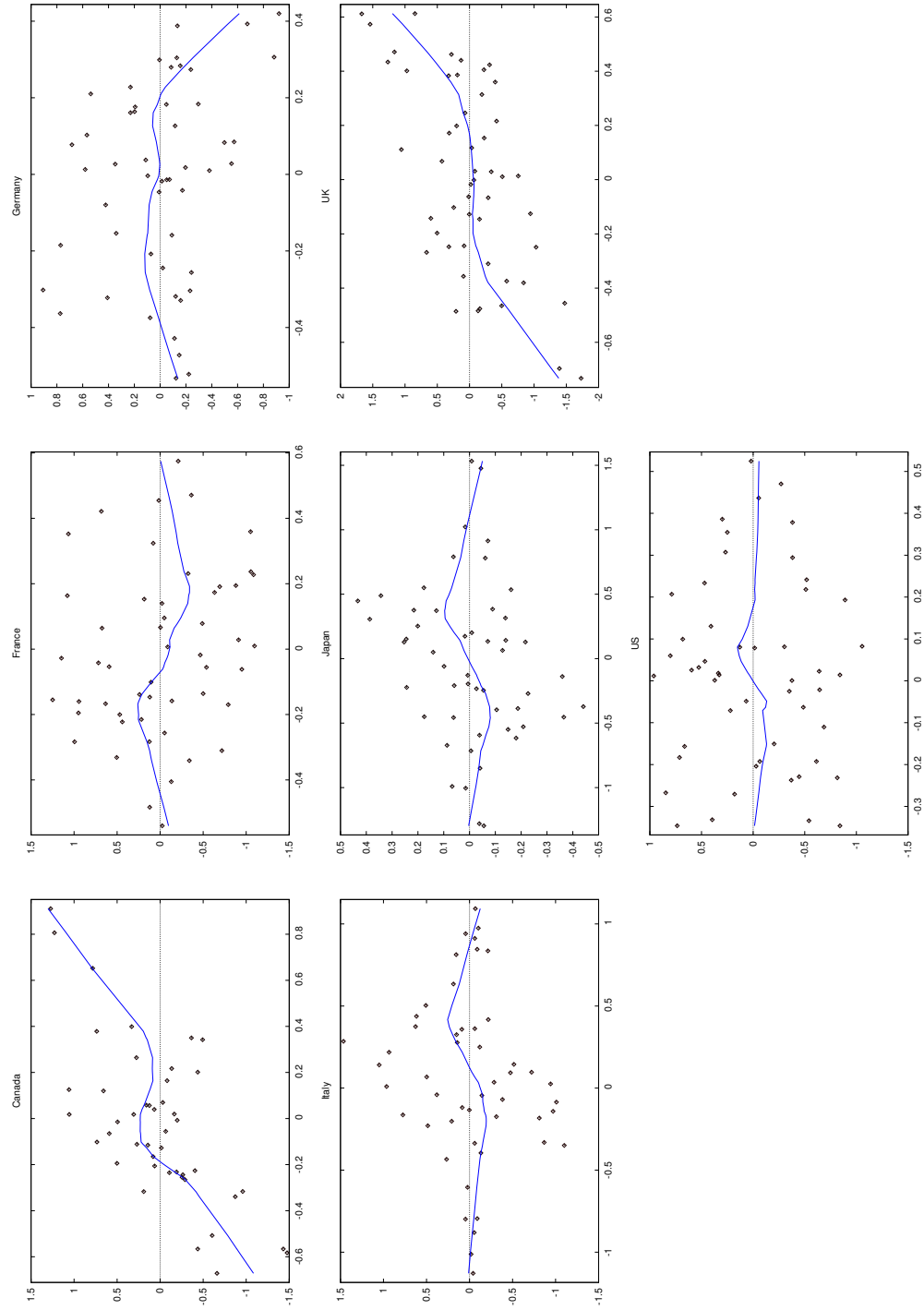


Figure 4: Scatter plot an loess fit (solid line) of labor productivity and unemployment rate for the D_3 components (8-16 years)

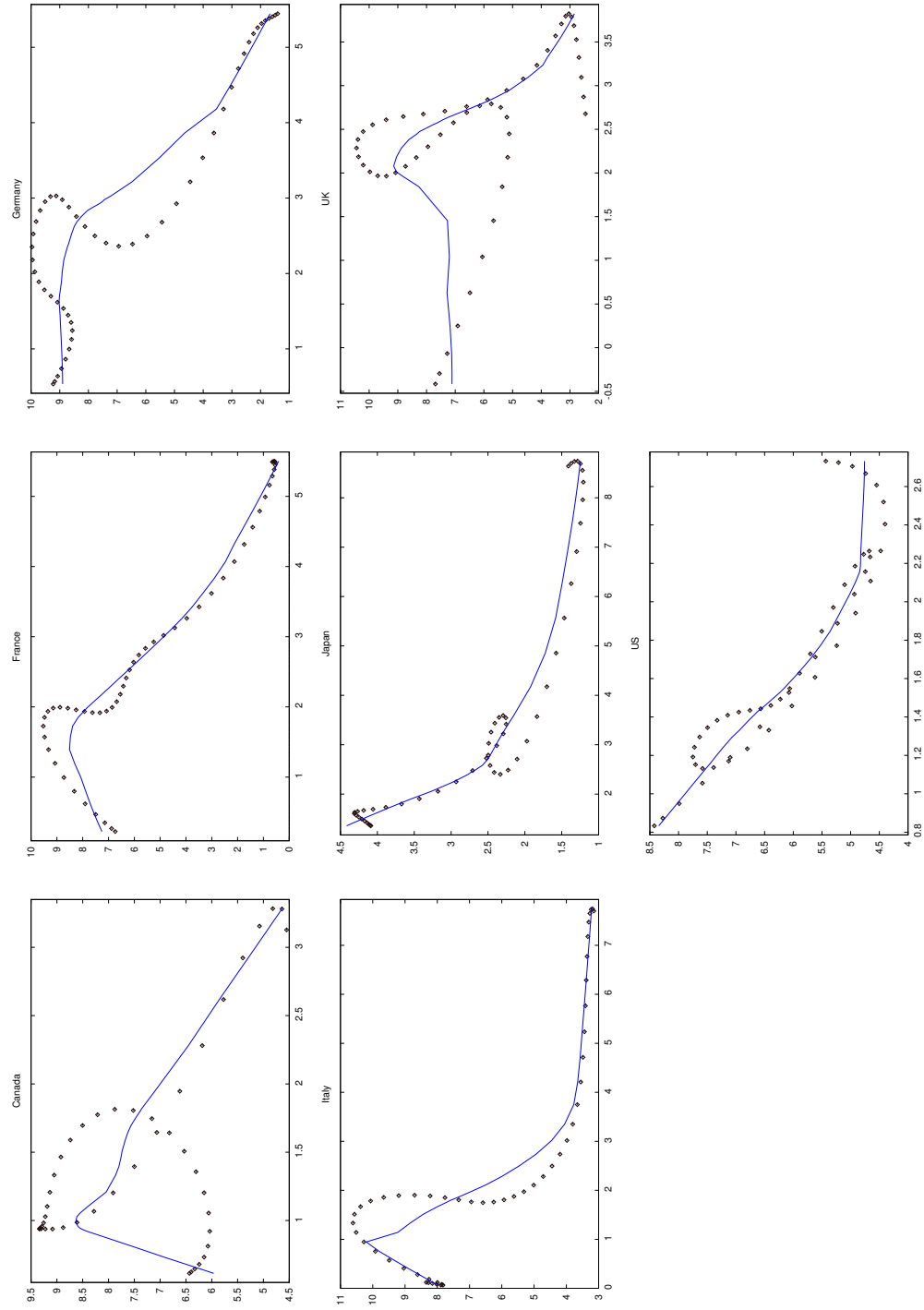


Figure 5: Scatter plot and loess fit (solid line) of labor productivity and unemployment rate for the S_3 long term components (> 16 years)

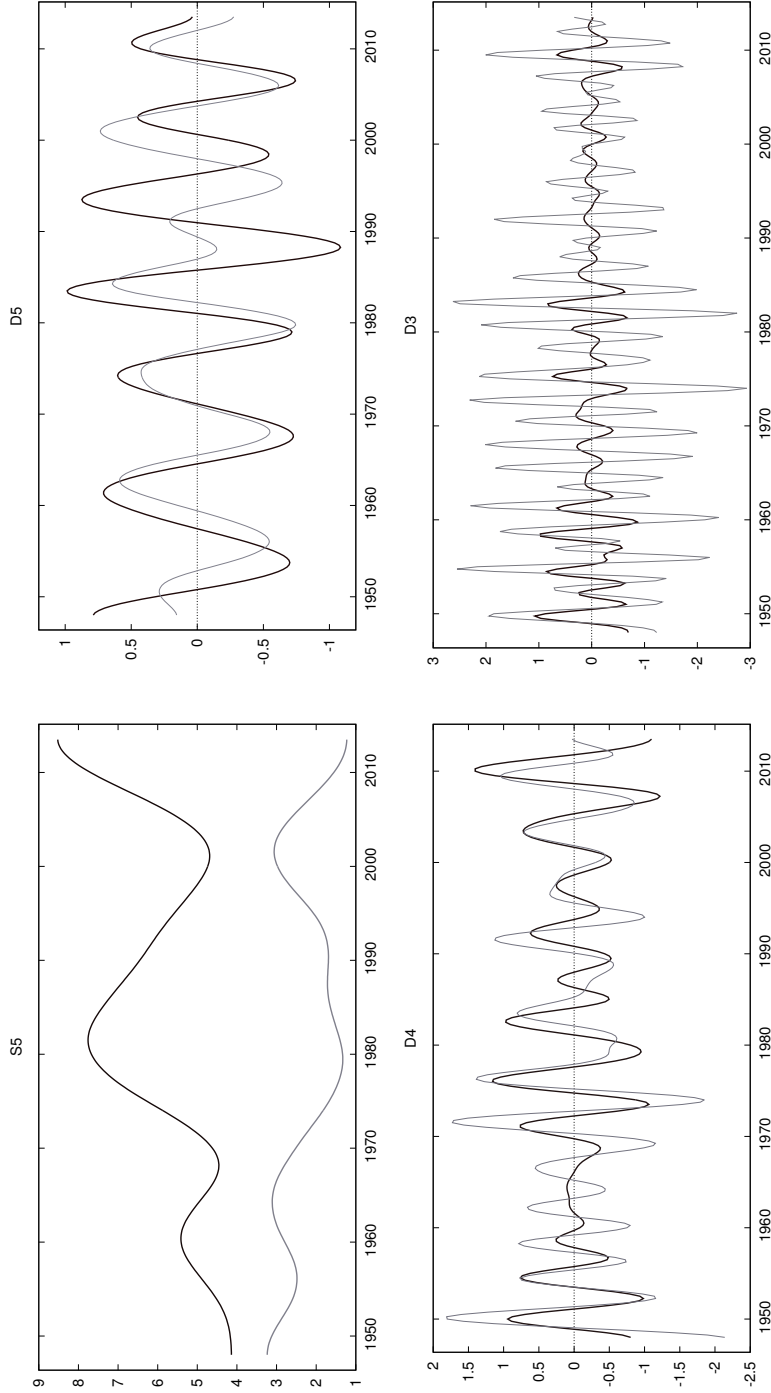


Figure 6: Phase shift relationships for unemployment (black thick lines) and productivity (grey thin lines) at different scale levels

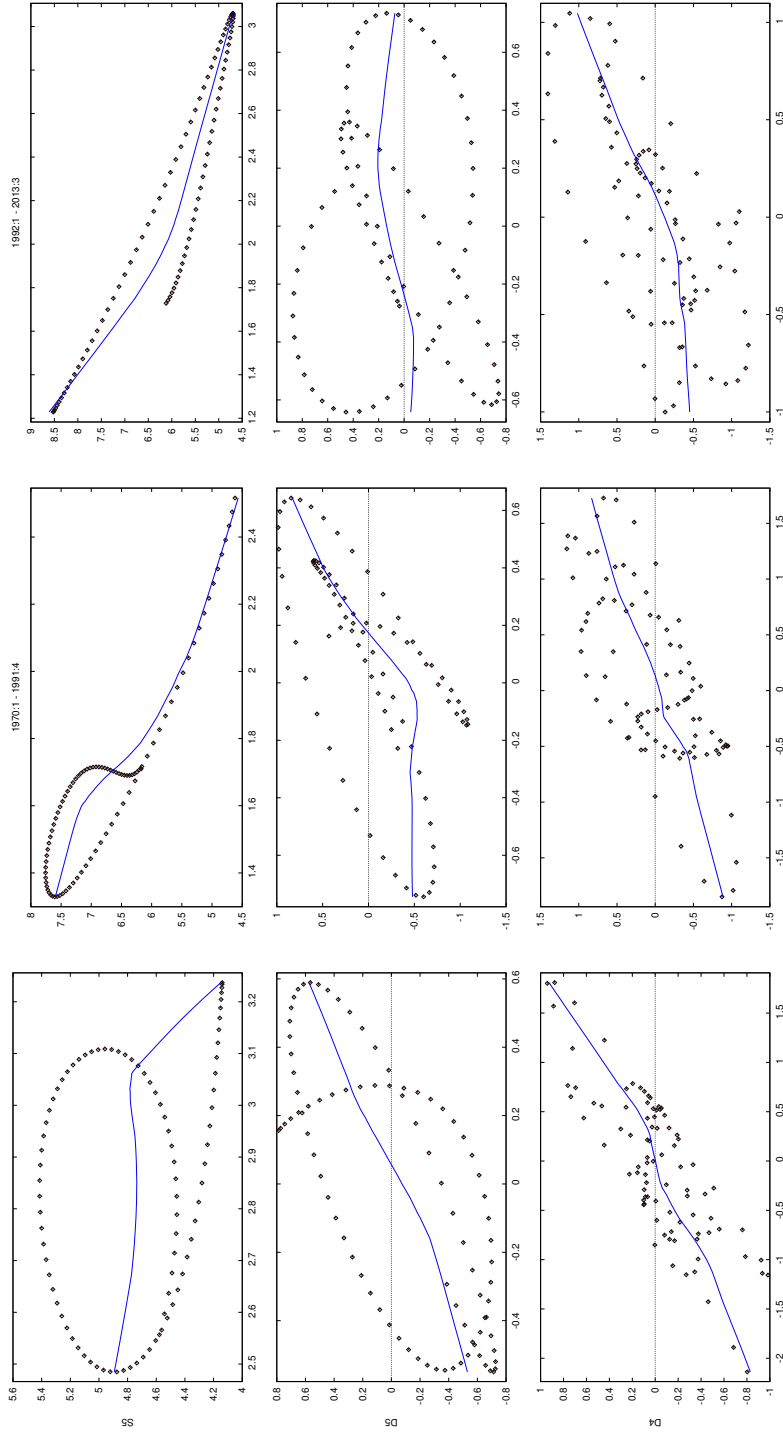


Figure 7: Scatter plot and loess fit (solid line) at different scale levels, S_5 (top row), D_5 (middle row) and D_4 (bottom row), and different time periods, 1948:2-1969:4 (left column), 1970:1-1991:4 (middle column) and 1992:1-2013:3 (right column).

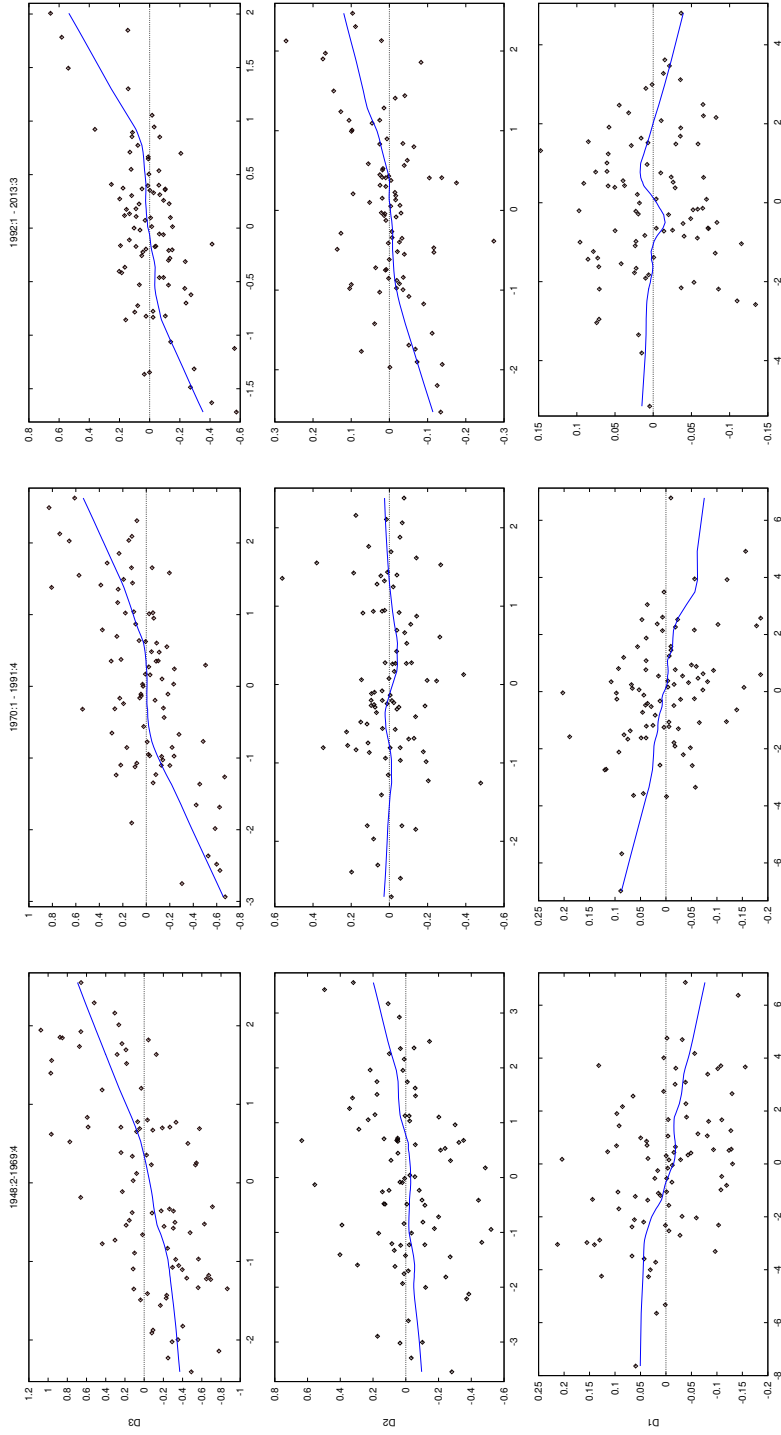


Figure 8: Scatter plot and loess fit (solid line) at different scale levels, D_3 (top row), D_2 (middle row) and D_1 (bottom row), and different time periods, 1948:2-1969:4 (left column), 1970:1-1991:4 (middle column) and 1992:1-2013:3 (right column).